A simulation of rill bed incision processes in upland concentrated flows

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\textbf{ABSTRACT}

Quantifying rill bed incision provides fundamental information for process-based erosion modeling; while the morphodynamic and hydrodynamic mechanism in bed incision processes are still unclear. Thus, experiments were conducted to examine rill bed incision processes in upland concentrated flows. DEMs (2 mm × 2 mm resolution) obtained by photogrammetry were used for rill bed morphology analysis. Rill channel (2.0 m-long, 0.08 m-wide and 0.15 m-deep) with two slope gradients (15° and 20°) were subjected to four overland flow rates (1.0, 2.0, 3.0 and 4.0 L min\textsuperscript{-1}). The results showed that sediment delivery, rill bed incision rate and average rill depth increased with inflow rate and bed slope. Sediment delivery increased from 0.060 to 0.226 kg min\textsuperscript{-1} per 1 L.min\textsuperscript{-1} inflow increment and from 0.043 to 0.207 kg min\textsuperscript{-1} when bed slope increased from 15° to 20°. In a well-developed rill channel, rill bed incision could be divided into three phases: pre-headcut formation (dominated by rill flow shear stress), headcut incision (dominated by headcut advancing) and post-headcut incision (dominated by rill flow shear stress). Headcut incision phase, which only accounted for < 15% of total experimental time, produced > 65% of rill bed sediment. In the pre-headcut formation phase, rill flow velocity, shear stress and stream power increased with increases of inflow rate and slope gradient. Conversely, flow velocity showed no evident trend with increased inflow rate and bed slope during headcut incision phase. Initial headcut advancing rate could be predicted by a non-linear function based upon soil characteristics, rill flow shear stress and headcut height. Sediment delivery showed a power function with the product of inflow rate and squared bed slope. Because rill bed incision is dominated by headcut advancement and incision, practices for controlling headcut initiation should be implemented to decrease hillslope soil loss.

1. Introduction

Rill erosion accounts for > 70% of total sediment yield on rill and inter-rill dominated areas (Shen et al., 2016; Xiao et al., 2017a). Sediment yield increases significantly after rill formation due to the increased concentrated flow depth, velocity and shear stress compared to inter-rill flow (Lal, 2002; Liu et al., 2011). Rill development is a complex physical process which includes one or more following sub-processes: headcut migration, bed incision and sidewall expansion (Meyer et al., 1975; Bingner et al., 2016). The dominant sub-process is different at different rill development stages (Shen et al., 2016). However, rill bed incision, depending upon landscape slopes and soil conditions, may play a more important role than sidewall expansion after headcut retreat (Bingner et al., 2016). Understanding the processes and mechanisms of bed scour and headcut migration can help improve the Water Erosion Prediction Project (WEPP) (Nearing et al., 1990; Zhu et al., 1995). Characterizing rill erosion processes and estimating their contributions, especially rill bed incision, on the Loess Plateau of China are important for understanding loessial rill erosion processes in this region (Shen et al., 2016).

Based on the assumption that rill cross-sectional area is constant (i.e. rill width/depth changes less than rill length), rill volume prediction equations have been fitted to rill length (Capra et al., 2009; Di Stefano and Ferro, 2011; Woodward, 1999). Liu et al. (2015) indicated that an approach including maximum rill depth and length was more accurate in predicting rill erosion than those solely based on rill length. Woodward (1999) pointed out that accurately estimating rill depth was a key factor to improving rill erosion model prediction accuracy. As a result, research on rill depth dynamics and its hydrodynamic mechanisms are of great importance to rill erosion modeling.

Currently, studies on rill bed incision have mainly focused on contributions of rill bed incision to soil loss (Shen et al., 2016), rill incision...
circularity (Bryan and Poesen, 1989; Bennett et al., 2000), affecting factors (e.g. soil erodibility, bed slope, upslope inflow rate, sediment concentration and near-surface hydraulic gradient) (Bryan and Poesen, 1989; Slattery and Bryan, 1992; Bennett et al., 2000; Alonso et al., 2002; Liu et al., 2015) and rill incision numerical modeling (Zhu et al., 1995; Favis-Mortlock, 1998; Casal et al., 2003; Jia et al., 2005). Evans and Boardman (2003) indicated that sufficient runoff volume and velocity were prerequisites for producing incisive flow that may lead to bed incision. Rill incision starts when concentrated flow tractive forces exceed rill bed soil entrainment resistance.

Rill depth does not increase uniformly with slope length as relatively narrow and deep rills (erosion zone) can alternate with straight and shallow rills (deposition zone), which may be attributed to the significant difference in erodibility between top soil (surface seal) and subsoil (Bryan and Poesen, 1989). Soil surface seal protects the rill bed from being eroded, which leads to low soil detachment rate of the rill bed by concentrated rill flow (Römkens et al., 1990). Failure of surface seals facilitates the formation of headcuts, bed incision and rill development (Römkens et al., 1990; Bennett et al., 2000). Headcuts are step changes in elevation that occur in channel networks (Gardner, 1983; Mosley, 1974). Based on the occurrence, position and time, Bryan and Poesen (1989) and Slattery and Bryan (1992) classified these headcuts into initial headcut and secondary headcut. However, it is hard to accurately measure rill depth variation and hydrodynamic characteristics near headcuts due to the rapid development of rills, narrow and deep rill morphologies and high sediment concentrations within rill flow (Römkens et al., 1990; Slattery and Bryan, 1992). Rill incision mechanism research, before and after surface seal failure, still needs to be intensified. It limits the establishment of soil erosion models for rill bed incision.

Rill morphology research based upon traditional surveying (rill survey with steel ruler, profilemeter, etc.), high precision GPS (RTK) and TLS (terrestrial laser scanning) have deepened the understanding of the magnitude of hillslope erosion (Rejman and Brodowski, 2005; Vinci et al., 2015; Shen et al., 2016; Vinci et al., 2016; Qin et al., 2017). However, manual measurements have some drawback including low outcome precision, low work efficiency, overestimates of rill depths and underestimates of rill widths et al. (Qin et al., 2016; Vinci et al., 2016). RTK and TLS greatly improved work efficiency and measuring accuracy in some extent while still have limitations caused by the quick changes of soil surface morphology (Mommm et al., 2015). Photogrammetry, based on the theory of remote-sensing imagery interpretation, provides a low-cost and labor-saving way to acquire a large number of points without disturbing the soil surface, with acceptable precision by using ordinary cameras or smartphones (Tarolli, 2014; Eltner et al., 2016; Wells et al., 2016; Qin et al., 2016; Vinci et al., 2017). The capability of photogrammetry (including Structure from motion photogrammetry and UAV) to produce high precision DEMs/DTMs that may be used to characterize erosion-deposition mechanics to provide new insight on monitoring rill morphology on movable soil beds (Berger et al., 2010; Gessesse et al., 2010; Tarolli, 2014; Bingner et al., 2016; Wells et al., 2016).

Concentrated-flow erosion is a major component of cropland erosion, and experiments that focus on specific processes are needed (Zhu et al., 1995). In this study, rill channel flumes were formed to investigate the effects of slope gradient and inflow rate on rill bed incision based on high precision DEMs obtained from photogrammetry. The specific objectives of this study were: 1) to detect rill depth variation with time and slope length under different experimental designs, 2) to discuss rill flow hydrodynamic characteristics during different rill bed incision phases, and 3) to establish predictive equations for headcut advancing rate and sediment delivery rate caused by bed incision.

![Fig. 1. a) Sketch of the experimental facilities and b) example of paired photos obtained from photogrammetry during experiment. 2015-12-19-11-16-11 represents year-month-date-hour-minute-second of the time the photo was taken. US and DS represent upstream and downstream, respectively.](image-url)
2. Materials and methods

2.1. Experimental materials and facilities

The soil used in this study was loess (fine-silty and mixed), which can be classified as a Calcic Cambisol (USDA NRCS, 1999). The top 20 cm of the Ap horizon was collected at a well-drained site in the Ansai County (36°45′N, 109°11′E), which is located in the hilly gully region of the Loess Plateau in Shaanxi Province, Northwest China. The soil texture was 28.3% sand (> 50 μm), 58.1% silt (50–2 μm), 13.6% clay (< 2 μm), and contained 5.9 g kg⁻¹ soil organic matter. Impurities, such as large organic matter and gravel, were removed from the soil, and the soil was passed through a 5-mm sieve prior to packing.

Four soil boxes measuring 200 cm-long, 30 cm-wide and 50 cm-deep (Fig. 1), containing drain holes (1 cm grid spacing) in the soil box bottom, were used in this study. To minimize the occurrence of systematic error, preparation of each soil box followed a same procedure and two soil boxes were selected randomly for each treatment with two replicates. Flow discharge was controlled by a constant-head water tank fixed 2.5 m above the soil box (Fig. 1).

2.2. Experimental design

This study consisted of 16 experimental runs, two factors were considered: four inflow rates (1.0, 2.0, 3.0, 4.0 L min⁻¹), which were equivalent to the rainfall intensities of 30, 60, 90, 120 mm h⁻¹ and accorded with rainfall standard that may cause intensive rill erosion on the Loess Plateau, Zhou and Wang, 1987) and two slope gradients (15° and 20°), which are the typical slope gradients on the Loess Plateau where rills develop, Zheng, 1989). The lengths of different experimental runs were in accordance with inflow rates to keep the total inflow volume constant (60 L). For 1.0, 2.0, 3.0 and 4.0 L min⁻¹ inflow rate, the durations of experiments were 60, 30, 20 and 15 min, respectively. Each experimental run included two replicates. Rill channel flume allows erosion with a maximum depth and width of 16 and 8 cm, respectively. During the experiment, two cameras were controlled by one infrared remote control at 30–60 s intervals. When these cameras received signal from the control, they shot simultaneously. The interval was reduced to 10 s during the process of headcut formation. A summary of experimental parameters is given in Table 1.

Table 1

<table>
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<th>Inflow duration/ min</th>
<th>Runoff rate/ L min⁻¹</th>
<th>Sediment delivery/ kg min⁻¹</th>
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2.3. Experimental procedures

2.3.1. Construction of the soil box

2.3.1.1. Rill channel flume construction. Two steel plates measuring 2 m were fixed at 11 and 19 cm widths of soil box to limit the rill widening process (Fig. 1). Sand paper (0.5-mm diameter sand was glued) was attached to the inner side of the steel plates to simulate grain roughness on rill sidewalls, and to reduce boundary effects. Rill channel flumes in this study were to simulate well-developed single rills after a headcut retreated upstream. Based on the definitions of Bryan and Poesen (1989) and Slattery and Bryan (1992), in this study, headcuts developed at the initial surface in the above-mentioned rill channel flumes were named with initial headcuts, and further developed headcuts at the downstream side of initial headcuts were named with secondary headcuts (Fig. 1).

2.3.1.2. Filling soil box. Sand was filled outside the steel plates in the soil box. The bottom 30-cm depth inside the steel plates was filled with sand while the top 20 cm was packed with soil in 0.05 m increments with a 1.10 g cm⁻³ soil bulk density (Fig. 1). A highly permeable cloth was used to separate the sand and soil layers. Each packed soil layer was lightly raked before placing the next layer for homogeneity. The amount of soil in each layer was kept as constant as possible to maintain a similar soil bulk density and a uniform spatial distribution. The transition section between the inlet tank and the rill channel flume was formed and stabilized with cement (Wells et al., 2009).

2.3.2. Physical experiment phases

1) After the preparation of the soil box, a 20-mm h⁻¹ pre-rain was applied for 30 min to the soil bed set at 3° slope gradient. This ensured that no water ponded on the soil surface. The purpose of the pre-rain was to develop consistent soil moisture, consolidate loose soil particles by raindrop impact, produce surface seal and reduce the spatial variability of underlying soil conditions. After a 12 h period of soil water redistribution, the bed slope was adjusted to the designed experimental slope gradient (15° or 20°) (Wells et al., 2009).

2) At slope lengths of 70 and 120 cm, two cameras (Canon EOS 5D Mark II) were mounted 1.5 m above and parallel to the centerline of the soil bed. Cameras were set to appropriate position which could ensure that photos were parallel to soil surface and overlapped completely. Cameras were fixed on a stable steel shelf and no tilt shift was applied during experiment. Then, adjustments were made to both cameras: 1) set shooting type as RAW with the highest camera resolution (2720 × 4080); 2) selected manual scene mode, set appropriate aperture (f/2.8), ISO (250) and shutter speed (1/20 s) and kept them constant during each run; 3) focused automatically until clear photos obtained, then set focus mode as manual focus (MF), focal length (24 mm) was kept constant during each run.

3) Similar to the method developed by Wells et al. (2013), four targets paralleled to the soil surface were setup at the boundary of the soil box (Fig. 1). Total station was used to measure the exact relative coordinates of each target.

4) Inflow rates were calibrated to the target flow rates (i.e. 1, 2, 3, 4 L min⁻¹) with a relative error smaller than 5% before running (Table 1). After feeding the concentrated inflow, the runoff and sediment from the test sample were captured in 5L plastic buckets at 30–60 s intervals, which was similar to the camera shooting interval. Rill flow width, depth and velocities were measured on 2 min intervals during the experiment. Flow velocity was measured by KMnO₄ dye tracing method at slope lengths of 40–90 cm and 110–160 cm. Flow depth and width were measured by gauge pin and steel ruler, respectively. All flow velocity data are the average value of three measurements multiplied by a coefficient of 0.75 (Shen et al., 2016).

5) After each run, runoff samples were left over night to settle so that the excess water could be decanted. The samples were then dried in an oven at 105 °C for 24 h and weighed to calculate runoff and soil loss.

2.4. Data analysis

Photos were imported into Agisoft Photoscan Professional 1.2.4 (Agisoft LLC, St. Petersburg, Russia) after each run. Dense point cloud
data were exported with .txt format after a series of preprocessing operations (including target detection, photo alignment, coordinate transformation and matching). The exported point clouds data were then imported into ArcGIS 10.4 (ESRI Inc., Redlands, CA, USA) to construct digital elevation models (DEMs). The process of DEM construction included: making an x, y event layer; creating a triangulated irregular network (Tin) and a fishnet of rectangular cells; performing spatial adjustment; recreating Tin; and Tin to raster. The interpolation used was natural neighbors. The resolution of DEM was averaged to a 2 mm × 2 mm raster grid.

Changes in average rill depth with time were obtained from a fishnet, created throughout the length of the rill channel (a series of parallel and equally spaced cross-section lines at 0.02 m) (Wells et al., 2013; Momm et al., 2015). Based on these high precision DEMs (Fig. 2), 78–83 rill depths were measured and averaged to the mean rill depth. Specifically, actual rill depth was calculated by the sum of water depth in rill and the depth calculated by DEMs. Changes of rill depth along the slope length (0–1.6 m slope length) were obtained by interpolation line drawn along the rill talweg where the profile graph of 3D analysis module of ArcGIS was used to detect the rill depth variation (Fig. 2).

Rill bed incision process is the work of runoff and the process of runoff energy dissipation. Runoff flow velocity (V), shear stress (τ) and stream power (ω) were selected to quantify rill depth variations at different rill bed incision phases.

1) Shear stress is calculated according to Eq. (1) (Foster et al., 1984):  
\[ \tau = \gamma RJ \]  
(1)

where \( \tau \) (Pa) is the shear stress, \( \gamma \) (g cm\(^{-1}\)) is the gravity of water, \( R \) (cm) is hydraulic radius, \( J \) (cm cm\(^{-1}\)) is hydraulic slope, which could be replaced by sine value of the angle (Zhu et al., 1995).

2) Stream power is calculated by Eq. (2):  
\[ \omega = \tau V \]  
(2)

where \( \omega \) (N cm\(^{-1}\) s\(^{-1}\)) is stream power, \( V \) (cm s\(^{-1}\)) is cross section mean flow velocity.

Equation fitting and validation were completed in Origin 9.0 (OriginLab Corp., Wellesley Hills, MA, USA). The determination coefficient (\( R^2 \)) and the Nash-Sutcliffe simulation efficiency (\( E_{NS} \)) (Nash and Sutcliffe, 1970) were used to evaluate the prediction accuracy of the fitted equations:

\[ R^2 = \frac{\sum_{i=1}^{n} (O_i - \bar{O})(Y_i - \bar{Y})^2}{\sum_{i=1}^{n} (O_i - \bar{O})^2 \sum_{i=1}^{n} (Y_i - \bar{Y})^2} \]  
(3)

\[ E_{NS} = 1 - \frac{\sum_{i=1}^{n} (O_i - Y_i)^2}{\sum_{i=1}^{n} (O_i - \bar{O})^2} \]  
(4)

where \( O_i \) is the observed value, \( Y_i \) is the predicted value, \( \bar{O} \) is the mean of the observed value, \( \bar{Y} \) is the mean of the predicted value, and \( n \) is the number of data. Typically, when \( R^2 > 0.6 \) and \( E_{NS} > 0.5 \), the equation prediction is acceptable or satisfactory (Santhi et al., 2001).

3. Results and discussion

3.1. Runoff and sediment process

Runoff rates were 3.7–7.1% lower than those of targeted inflow rates, which can be attributed to the water infiltrated through the soil.
and sand layer (Table 1, Fig. 3a, b). Sediment delivery increased with the increase of inflow rate and bed slope. Under a given bed slope, sediment delivery increased by 0.060 to 0.226 kg min\(^{-1}\) (41.7\%–122.1\%) when inflow rate increased by 1 L min\(^{-1}\). Under a given inflow rate, sediment delivery increased by 0.043 to 0.207 kg min\(^{-1}\) (44.2\%–83.1\%) when bed slope increased from 15° to 20°.

Sediment delivery increased with time while it showed different trends under different treatments (Fig. 3c, d). When inflow rates were 1 L min\(^{-1}\), sediment delivery increased with fluctuations and showed a step-like pattern. When inflow rates were equal to or > 2 L min\(^{-1}\), sediment delivery surged up at the beginning of experiment and then fluctuated around a high level with a tendency to decrease. The peak in sediment delivery occurred earlier with increased inflow rate. The peak occurrence times were 15.0–60.0 and 10.5–60.0 min under 15° and 20° bed slope, respectively; and the highest sediment deliveries were 0.283–0.797 and 0.289–1.000 kg min\(^{-1}\), respectively, with 1–4 L min\(^{-1}\) inflow rates.

3.2. Rill bed incision process

3.2.1. Time series of average rill depth

Average rill depth overall increased with time (Fig. 3e, f). Similar to sediment delivery, average rill depth presented a step-like pattern under lower slope and inflow rates at the beginning of the experiment. As inflow rate and slope increased, the duration of the plateau pattern became shorter. When inflow rate was lower than 2 L min\(^{-1}\), lower inclination of average rill depth trend line was due to a slower advancing rate of the initial headcut moving upstream and lack of new secondary headcuts. As a result, the overall bed incision rate was small.
When inflow rate was equal to or > 2 L min\(^{-1}\), rill depth increased linearly, which could be attributed to the continuous advance and incision of headcuts. With the inflow rates of 1, 2, 3 and 4 L min\(^{-1}\), the final average rill depths were 8.0, 10.1, 11.0, 11.2 cm for 15° bed slope and 9.0, 10.5, 11.2, 12.9 cm for 20° bed slope.

3.2.2. Variation of rill depth along slope length

Curves of rill depth versus slope length were plotted in Fig. 4 based on the high precision DEMs (Fig. 2) to study the rill bed morphology changes with time. At lower bed slope and smaller inflow rate (Run 1, 2, 5), rill bed morphology basically kept unchanged before and after the initial headcut moved upstream in the condition of no secondary headcut formation (Fig. 4a, b). In terms of higher inflow rates and larger bed slope (Run 3, 4, 6, 7, 8), secondary headcuts widely developed after initial headcuts passed through, which facilitated increased rill bed incision rate. After the headcut erosion, sediment deposited downstream of the headcut (Fig. 4c).

Following are the features of rill depth variation with time in a well-developed rill channel:

1) Rill bed incision caused by rill flow shear stress was low and homogeneously distributed along slope length before initial headcut formation;
2) Height of initial headcut remained unchanged after a short duration from the beginning of experiment. This phenomenon fitted well with the results given by Bennett et al. (2000) when the Typic Paleudult soil (sandy clay loam) was used while a little different from Lewis (1944), who pointed out that headcut height decreased with migration distance when conducting a headcut migration experiment with non-cohesive sand. The above comparisons indicated the impacts of soil intrinsic properties on headcut morphology.
3) Initial headcuts migrated at a constant rate under the same inflow rate and bed slope (Fig. 4a, b). Similar results were obtained by Bennett et al. (2000) when conducting a headcuts migration experiment with upland concentrated flow.
4) Secondary headcuts developed after initial headcuts moving upstream and showed different erosion-deposition characteristics (Fig. 4c). A bed slope reduction downstream of a migrating headcut was observed.
5) The spacing between two initial headcuts decreased with increased bed slope and inflow rate, which might be explained by the threshold distance for the roll wave development and the related
increase in shear stress.

3.3. Three phases of rill bed incision

Four typical cross-sections were selected to study the rill bed incision and mechanisms. The selection basis was: 1) the distance from selected cross-sections to transition section or rill channel outlet should be > 30 cm to alleviate the boundary effect; 2) the period with low bed incision rate at the beginning of experiment should be long enough to fully show the whole process of rill bed incision; 3) the presence of at least one initial headcut pass through the selected cross-section. The selection basis was: 1) the distance from selected cross-sections to transition section or rill channel outlet should be > 30 cm to alleviate the boundary effect; 2) the period with low bed incision rate at the beginning of experiment should be long enough to fully show the whole process of rill bed incision; 3) the presence of at least one initial headcut pass through the selected cross-section.

Rill bed incision rate versus time presented a low fluctuation – surge – decline with fluctuation trend (Fig. 5). The number of bed incision peaks, the duration of low fluctuation phase and the duration of the peak in incision rate varied under different inflow rates and bed slopes. In regard to a certain cross-section, rill bed incision might be divided into three phases in a well-developed rill channel:

1) Pre-headcut formation phase, lasted until initial headcut formation, was dominated by rill flow shear stress erosion. Soil particles were slowly and homogeneously entrained by rill flow. Rill bed incision detachment rate was determined by flow velocity, hydraulic radius and hydraulic slope. Rill bed incision rate varied between 0.1 and 4.7 mm min⁻¹, which was the lowest among three phases (Fig. 5). It corresponded well to the results of Bennett et al. (2000), who indicated that sediment yield from soil surface was nearly zero prior to bed incision.

2) Headcut incision phase, the most important phase of rill bed incision, caused the largest rill bed incision rate (11.5–90.1 mm min⁻¹), which exhibited 19–115 times those of the previous phase (Fig. 5). Soil surface seal formed during rainfall pre-wetting was likely to break at weaker points due to the entrainment of soil particles by flow shear stress and result in the exposure of underneath highly erodible soil. The soil was then entrained quickly by rill flow without the protection of surface seal, and as a result, initial headcuts formed quickly and began a new round of headcut advance. During the headcut advancing process, the scour hole depth increased with increased inflow rate, which verified the results given by Wells et al. (2009). Rill flow dropped off the headcut, where, the jet flow, caused by the transformation of potential energy into kinetic energy, was the main power to scour soil immediately below the headcut. When the soil gravity or shear force exceeded the soil particle cohesive force, rill head collapsed (or caved in) promoting headcut advance. Also, the duration of the headcut incision phase was composed of 5.0%–13.3% of total experiment time (the shortest duration among three phases) while rill bed sediment produced during this phase accounted for 65.0%–86.2% total sediment delivery.

3) Post-headcut incision phase, dominated by rill flow shear stress, exhibited a bed incision rate of 2.0–16.4 mm min⁻¹ (Fig. 5). Without the protection of surface seal, the rill bed was directly exposed to concentrated flow. However, rill flow energy was dissipated by forming roughness created by headcut advancing upstream and by transporting sediment. Also, as rill developed and widened, the flow depth reduced, which lowered flow shear stress and stream power. Therefore, rill bed incision rate was relatively low due to reduced flow energy, which was not enough to produce further incision.

The above results supported the conclusions given by Zhu et al. (1995), who indicated that erosion due to bed scour below the headcut constituted only 2% of total detachment. Jia et al. (2005) indicated that there was a threshold for downstream channel erosion occurrence below the headcut when headcut migration was simulated by a numerical model. Bryan and Poesen (1989) and Slattery and Bryan (1992) pointed out that rill bed incision is always accompanied by headcuts formation and development, which has been proven by this study. However, research findings of this study also indicated that bed incision detachment caused by rill flow shear stress before and after headcut incision, though very slight, cannot be ignored.
Hillslopes (e.g. intercepting runoff) determined by secondary headcut incision process. Therefore, rill bed incision always goes with the new headcut formation and advance. Processes during headcut incision phase. The most active phase of rill bed incision is evident during the headcut incision phase or the post-headcut formation phase. Correspondingly, sediment delivery and rill bed incision rate varied little.

During the headcut incision phase, rill bed morphology and hydrodynamic indicators changed significantly, rill flow resistance force increased. One or more headcuts occurred on the measured cross-sections (Fig. 4). Incision depth increased with increased slope and inflow rate. Vortex occurred when rill flow passed through headcut. Rill flow velocity did not rigorously increase with increased inflow rate due to the increased form roughness caused by headcuts. Rill flow velocity significantly decreased compared to the previous phase. Rill flow was confined into a narrow flow path on downstream of headcut which led to decreased flow width and increased flow depth. As a result, flow shear stress and stream power increased. Correspondingly, rill incision rate showed sharp peaks while the inclination of rill depth-time trendline and sediment delivery increased (Fig. 3).

During post-headcut incision phase, rill flow flowed in a reshaped and somewhat widened incised-flow path with high bed roughness. Large amount of rill flow energy was consumed by headcut advance upstream and sediment transportation. As a result, rill flow velocity was lower while rill flow shear stress was higher than those of the pre-headcut formation phase due to a more concentrated flow condition (flow width ranged 0.9–3.6 cm, which was 2.9–4.2 cm narrower than those of the pre-headcut formation phase; flow depth ranged 0.5–4.0 cm, which was 0.4–3.6 cm deeper than those of the pre-headcut formation phase). Compared to the headcut incision phase, stream power as a function of flow velocity and shear stress, was slightly smaller at higher flow rates.

3.5. Rill bed incision equations fitting and validation

Initial headcut advancing rate, to some extent, is a determining factor in rill bed incision process. Based on the discussion above, three main factors showed significant impacts on initial headcut advancing process: soil property, flow energy and headcut height. Values used to fit the advancing equation are provided in Table 2. Rill flow characteristics (shear stress/stream power) were influenced by upslope inflow rate and bed slope. The concentrated flow shear stress ($\tau$) immediately above the headcut, which was greater than the soil critical shear stress, facilitates the entrainment of soil particles by rill flow. Headcut height ($h$), which directly reflected the gravitational erosion during the headcut advancing process, increased with the increase of inflow rate and bed slope, was chosen as another factor in the fitted equation (Bennett and Casali, 2001; Babazadeh et al., 2017). To take the intrinsic characters of the Loess soil into account, a coefficient $\beta$, representing soil erodibility and critical shear stress (the ability to resist gravitational collapsing and rill flow entrainment), was used. It might be related to soil clay content and has been used to predict headcut advance and widening process in some soil erosion prediction models (Gordon et al., 2007). Following is the basic form of the equation:

$$F(V) = \beta \times f(\tau, h)$$

where $\tau$ (Pa) is concentrated flow shear stress, $h$ (cm) is average height of headcuts, $V$ ($\text{cm s}^{-1}$) is the average headcut advancing rate, $\beta$ is a coefficient representing soil erodibility and critical shear stress. To confirm the final shape of Eq. (5), separate equations representing the
relationships between impacting factors (τ, h) and V were formulated. The results showed that power functions could be used to simulate V by τ and h. Therefore, F(V) may be expressed as a non-linear equation:

\[ V = 1.395 \times (\tau^{0.302} + h^{0.367}) \]  

This relationship is sediment delivery, and is not considered in some rill erosion formulas (Wirtz et al., 2012). Therefore, Eq. (6) may provide some basis for rill depth prediction when the following three factors were included: soil erodibility, concentrated flow character and gravitational impacts. Eq. (7) is suitable for the prediction of sediment delivery caused by rill bed incision of a single rill developed on loessial hillslope. These results also verified that sediment delivery caused by rill erosion increased with bed slope and inflow rate and, showed significant relationship with the combinations of Q and S (Meyer et al., 1975; Slattery and Bryan, 1992; Zhu et al., 1995; Berger et al., 2010).

4. Conclusions

Simulated upslope runoff inflow experiments that focused on rill bed incision were conducted under four inflow rates (1, 2, 3, 4 L min⁻¹) and two slope gradients (15° and 20°). Variations of rill depth in time and slope length series were quantified. Rill incision phases, corresponding hydrodynamic characteristics and fitted equations of headcut advancing rate and sediment delivery rate were discussed. The following observations were made.

1) Sediment delivery, final average rill depth, rill flow shear stress and stream power increased with increases of inflow rate and bed slope.
2) Pre-headcut formation phase exhibited a low rill bed incision rate, which could be attributed to low rill flow shear stress and stream power in spite of a relatively high rill flow velocity.
3) Headcut incision phase exhibited 19–115 times the bed incision rate of the pre-headcut phase due to headcut advances and increased rill flow shear stress and stream power.
4) Without the protection of soil surface seal, post-headcut incision phase exhibited a moderate bed incision rate compared to the other two phases.
5) Headcut advancing rate could be predicted by a non-linear function depend upon soil erodibility, concentrated flow characteristics and headcut height. Sediment delivery was expressed as a power equation to \( Q^{2} \).

Rill bed incision is largely determined by headcut formation and development while the incision caused by rill flow shear stress before and after headcut incision, though very slight, cannot be ignored. Practices on preventing headcut formation should be emphasized to control rill initiation and formation.

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