Using the double-exponential water retention equation to determine how soil pore-size distribution is linked to soil texture

Dianyuan Ding\textsuperscript{a,b}, Ying Zhao\textsuperscript{a,c,d,*}, Hao Feng\textsuperscript{a,e,*}, Xinhua Peng\textsuperscript{f}, Bingcheng Si\textsuperscript{a,b,d}

\textsuperscript{a}State Key Laboratory of Soil Erosion and Dryland Farming on the Loess Plateau, Northwest A&F University, Yangling 712100, China
\textsuperscript{b}College of Water Resources and Architecture Engineering, Northwest A&F University, Yangling 712100, China
\textsuperscript{c}Key Laboratory of Plant Nutrition and the Agri-environment in Northwest China, Ministry of Agriculture, Northwest A&F University, Yangling 712100, China
\textsuperscript{d}Department of Soil Science, University of Saskatchewan, Saskatoon, SK, S7N 5A9, Canada
\textsuperscript{e}National Engineering Research Center for Water Saving Irrigation at Yangling, Yangling, Shaanxi 712100, China
\textsuperscript{f}State Key Laboratory of Soil and Sustainable Agriculture, Institute of Soil Science, CAS, Nanjing, China

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The link between soil particle arrangement and pore structures and its role in soil water retention is important. However, it is not completely determined. In this study, based on similarities and synchronizations between the three terms, i.e., residual, matrix, and structural porosities in a double-exponential water retention equation (DE) and the three soil separates (clay, silt and sand), we aimed to explore the link between soil particles and pores and to determine the causes of mono- and bi-modality in DE differential functions. For this purpose, 78 soil samples were selected from the Unsaturated Soil Hydraulic Database with a wide range of soil texture. Results showed that the DE fitted with the measured soil water retention curves well. Soil residual and structural pores could be reasonably predicted with clay and sand contents, respectively, via the derived linear functions. Soil texture reasonably influenced the modality of DE differential functions, particularly in the following situation. (1) When a soil separate (e.g., silt) occupied the absolute majority in soil particle size distribution, the DE differential function may tend to show a mono-peak, whereas the others may tend to be bi-peak. (2) On the contrary, the matrix peak was significantly influenced by clay content, and the structural peak was greatly affected by sand content. (3) When the sand content was greater than 0.5, the curve of the DE differential function may tend to be bi-peak, i.e., a high structural peak together with a low matrix peak. Further analyses, e.g., the technologies of X-ray and original SEM are needed to confirm the proposed soil structural model.

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1. Introduction

Soil structure has traditionally been considered as one of the dominant attributes of soil quality, which critically influences the hydraulic properties of unsaturated media, such as soil water retention curves (WRCs; Nimmo, 1997). Soil macromorphology and micromorphology are closely linked to soil structure, and they are considered the basis of the explanation about the influence of soil structure on soil hydraulic functions (Kutilek, 2004). Although the processes that relate soil water fluxes to the macro- and micromorphological characteristics of soil structure have been identified (Lin et al., 1999; Vervoort and Cattle, 2003), the quantitative relationships between the morphologic characteristics of soil structure and soil hydraulic functions remain unclear.

In recent years, the soil particle size distribution (PSD) has been widely used to estimate the soil WRC. Arya and Paris (1981) developed a semi-empirical model (i.e., AP) to predict WRCs based on the similarity between shapes of the cumulative PSD and WRC. The AP and the modified AP model (Arya et al., 1999a,b) are most applicable to sandy soils (Hwang and Choi, 2006; Hwang and Powers, 2003). Havercamp and Parlange (1986) estimated WRCs directly from the PSD data by assuming a lognormal distribution for both PSD and pore size distribution (POD) in sandy soils. Kosugi (1994) applied a three-parameter lognormal distribution function to the PODs of soils and developed a soil water retention model, which was further modified in his work in 1996 to be compatible with the model of Mualem (1976). The two lognormal functions described earlier are applicable to most soils. Moreover, many

\textsuperscript{*} Corresponding authors at: Institute of Soil and Water Conservation, Chinese Academy of Sciences, No.26 Xinong Road, Yangling, Shaanxi 712100, China. Fax: +86 029 87012210.

E-mail addresses: yzhaosols@gmail.com, yzhao@nwsauf.edu.cn (Y. Zhao), nerccwi@vip.sina.com (H. Feng).

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pedo-transfer functions based on soil texture have also been developed to predict WRCs (Botula et al., 2012; Hodnett and Tomassella, 2002; Oliveira et al., 2002).

Symmetrical or asymmetrical relationships may exist between the PSD and the POD of soils. Hwang et al. (2011) defined that the symmetry between PSD and POD means that the shape of POD curve is symmetric with respect to PSD curve. Therefore, POD curve can be superimposed on PSD curve except that the shape of POD is asymmetric to that of the PSD. Some researchers attempt to derive fractal WRC functions based on the symmetry of the relationship and most of their functions can be calculated from the POD. Tyler and Wheatcraft (1990) introduced a fractal model for WRCs with the Sierpinski carpet model, which provides a theoretical basis for the Brooks and Corey (1964) model. Perrier et al. (1999) and Bird et al. (2000) developed a pore-solid fractal (PSF) model to estimate WRCs from the cumulative PSD data based on the assumption that the soil structure shows self-similarity. In addition, Hwang et al. (2011) modified the PSF model by considering the asymmetry between PSD and POD; the model hypothesizes that the microscopic arrangement of soil particles affects pore geometry and POD.

Many WRC functions, such as that of Rieu and Sposito (1991a,b), may perform well on some data sets, but not on others (e.g., dry range). This problem may arise from the insufficient attention that is paid to textural effects, which may dominate water retention beyond the wet range (Nimmo, 1997). Nimmo (1997) partitioned the pore space into two components, namely, textural and structural to model WRCs. On the basis of the laws of hydrostatics and hydrodynamics, Kutílek (2004) made a detailed classification of soil pores, namely, submicroscopic pores, micro-pores and macro-pores, where micro-pores include matrix pores (textural component) and structural pores. However, the structure of soil pores was mainly influenced by the soil solid phase. Moreover, how the soil separates (i.e., clay, silt and sand) affect these different soil pores is still unclear.

Dexter et al. (2008a) suggested that the compound particles hierarchy has three levels: groups of primary particles comprising micro-aggregates, groups of micro-aggregates comprising aggregates, and groups of aggregates comprising bulk soil. Every level of particles has a corresponding level of pores whose radii increase with the radii of particles. Accordingly, a pore space can be considered in terms of the void ratio (\(e_{\text{total}}\)), which consists of residual pores (\(e_{\text{residual}}\)), matrix pores (\(e_{\text{matrix}}\)), structural pores (\(e_{\text{structural}}\)), and macro-pores (\(e_{\text{macro}}\)). It is expressed as follows:

\[
e_{\text{total}} = e_{\text{residual}} + e_{\text{matrix}} + e_{\text{structural}} + e_{\text{macro}}
\]

(1)

For normal water retention experiments, only \(e_{\text{matrix}}\) and \(e_{\text{structural}}\) are considered, and the segregation of pore space into these two categories produces what are called bi-modal PSDs (Dexter et al., 2008a; Omuto, 2009). The double-exponential water retention equation (DE) may then be expressed as follows:

\[
w = C + A_1 e^{-(h/h_1)} + A_2 e^{-(h/h_2)}
\]

(2)

where \(w\) is soil volumetric water content; \(C\) is residual porosity i.e., residual water content defined as the water fraction that remains in the soil as the applied suction increases toward infinity (the asymptote of the equation, Dexter et al., 2008a); \(A_1\) and \(A_2\) are matrix and structural porosity, respectively; \(h_1\) and \(h_2\) are the characteristic pore saturations at which the matrix and structural pore spaces, respectively, are empty. Therefore, the second and third terms in Eq. (2) represent the variation in matrix and structural pore space with changing pore suction. DE not only provides a good fit to water retention data, but also accounts for how soil bulk density increases at the expense of structural porosity (Dexter et al., 2008a). Additionally, DE has been used to study the influence of the complex organic carbon on the smallest soil pores (residual plus textural) (Dexter et al., 2008b). The saturated hydraulic conductivity (Dexter and Richard, 2009a) and the optimum water content for tillage (Dexter and Richard, 2009b) have also been studied.

Interestingly, the DE partitions the corresponding soil pores into three components. Precise definitions are presented. The pore of \(C\) is so small that it may not be characterized in standard water retention experiments, which usually stop at the suction of 15,000 cm for WRC and not toward infinity. \(A_2\) is the pore space among individual soil mineral particles. \(A_2\) is the pore space among the micro-aggregates and among incipient aggregates. However, these definitions are predefined empirically. Moreover, a minor confirmation exists in terms of the physical meanings of DE parameters.

The differential function of WRC is related to the POD function or pore capillary pressure distribution function (Kosugi, 1994). Soil POD can be reflected by the mono-modality, bi-modality or even tri-modality (Dexter and Richard, 2009b; Dexter et al., 2008a; Kutílek, 2004; Kutílek et al., 2006). However, the causes for either the mono- or bi-modality of DE differential function are poorly understood. We speculate that the various DE differential functions, i.e., mono- or bi-modality, can be explained by using the soil texture and whether or not the three soil pore types established by the DE can be linked to the soil separates (i.e., clay, silt, and sand). Based on soil samples with a wide range of soil textures selected from UNSODA, we aimed (i) to explore the relationships between the three soil separates and the three soil porosities (i.e., residual, matrix, and structural porosities) based on DE; (ii) to use soil texture explaining the different modalities of DE differential curves.

2. Theory

2.1. Pore classification by soil texture

In this study, we mainly focused on the packing pattern of soil particles. We considered that the packing pattern of soil aggregates be similar with that of soil particles. We also assumed that soils are formed naturally and have no artificial compaction. Pore spaces can be understood well when the soil structure is considered a hierarchy of compound particles (Dexter, 1988). Accordingly, the idea of pore classification based on soil texture emerged from the observed similarities between the three parameters \(C\), \(A_1\), and \(A_2\) of the DE (i.e., residual, textural, and structural pores) and the three

![Fig. 1. Packing pattern of soil particles, where the mean size of the pores around particles is assumed to increase progressively with the particle size.](image-url)
soil separates (i.e., clay, silt, and sand). The packing pattern of the soil particles influenced soil bulk density, POD, and pore continuity (Lal and Shukla, 2004). All the soil particles were assumed to be nearly spherical, and the mean size of the pores around particles progressively increased with the particle size (Fig. 1).

Haverkamp and Parlange (1986) suggested that symmetry exists between PSD and POD as mentioned earlier (e.g., Fig. 2). The pore radius of the i-th fraction \( r_i \) (cm) can be expressed as follows:

\[
 r_i = \frac{d_i}{\gamma}
\]

where \( d_i \) (mm) is the corresponding particle diameter; and \( \gamma \) is the packing-related parameter of the soil (mm cm\(^{-1}\)) ranging from 0.1 to 100 (Hwang and Choi, 2006; Hwang et al., 2011).

According to the soil texture classification triangle of the US Department of Agriculture, soil particles are generally partitioned into three separates, as follows: clay \((d_i < 0.002\) mm\)), silt \((0.002 < d_i < 0.05\) mm\)), and sand \((0.05 < d_i < 2\) mm\)). Based on our aforementioned assumption of pore classification, soil pores can also be partitioned into three classes through Eq. (3), and these classes correspond to the three soil separates, as follows: the pores around clay particles \( r_i < 0.002/\gamma \) mm\)), the pores around silt particles \((0.002/\gamma < r_i < 0.05/\gamma \) mm\)), and the pores around sand particles \((0.05/\gamma < r_i < 2/\gamma \) mm\)). The pores around clay, silt, and sand may represent the residual pore space \( C \), the matrix pore space \( A_1 \), and the structural pore space \( A_2 \), respectively. This assumption is similar to the pore classification of Kutílek (2004) and Dexter et al. (2008a). Therefore, we redefined the physical meanings of parameters \( C, A_1, \) and \( A_2 \) of the DE on the basis of soil texture.

### 2.2. Boundaries of pore spaces

As mentioned earlier, parameters \( h_1 \) and \( h_2 \) (cm) are the characteristic pore suctions where the matrix and structural pore spaces are empty. By assuming perfectly wettable surfaces \((\cos \theta = 1\), where \( \theta \) is the contact angle\), suction head \( h_i \) (cm) can be associated with pore radius \( r_i \) (cm) with the following equation (Brutsaert, 1966; Kosugi, 1996):

\[
 r_i = \frac{0.149}{h_i}
\]

Eq. (4) can transform parameters \( h_1 \) and \( h_2 \) into two boundaries of pore sizes. Based on the soil pore classification, we assumed that \( h_1 \) was the boundary between residual and matrix pores classes, whereas \( h_2 \) was the boundary between matrix and structural pore spaces. Therefore, the soil pore classification according to soil texture was established based on DE by dividing the soil pore into residual, matrix, and structural pores, and by considering \( h_1 \) and \( h_2 \) as the corresponding pore division points.

### 3. Materials and methods

A total of 78 soil samples representing a wide range of soil textures were selected from the Unsaturated Soil Hydraulic Database (UNSODA; Table 1; Leij et al., 1996). The selected data consisted of WRCs from undisturbed soil samples composed of at least 10 data points over a large suction scale (suction > 10 cm), most of which were measured with a pressure plate in the laboratory. In addition, most of the soil total porosity values were determined from soil bulk density and particle density. The soil texture of the samples was shown in the soil texture triangle.
Table 1
Basic information of the selected soil data from UNSODA.

<table>
<thead>
<tr>
<th>Texture</th>
<th>Number of data sets</th>
<th>UNSODA code</th>
<th>Clay</th>
<th>Silt</th>
<th>Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mean SD</td>
<td>Mean SD</td>
<td>Mean SD</td>
</tr>
<tr>
<td>Clay</td>
<td>6</td>
<td>1181, 1182, 2360, 2361, 4680, 4681</td>
<td>0.510</td>
<td>0.044</td>
<td>0.317</td>
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<tr>
<td>Clay</td>
<td>3</td>
<td>1180, 2740, 2743</td>
<td>0.329</td>
<td>0.036</td>
<td>0.346</td>
</tr>
<tr>
<td>Loam</td>
<td>17</td>
<td>1370, 2530, 2531, 2580, 2600, 2601, 2602, 2603, 2604, 2605, 2650, 2651, 2652, 2741, 2742, 2750, 2752</td>
<td>0.186</td>
<td>0.042</td>
<td>0.378</td>
</tr>
<tr>
<td>Silt loam</td>
<td>27</td>
<td>1280, 1281, 1331, 1490, 2001, 2002, 2010, 2011, 2012, 2581, 2670, 2671, 2710, 2760, 2761, 3360, 3361, 3370, 3371, 3380, 3392, 3393, 4510, 4530, 4671, 4672, 4673</td>
<td>0.164</td>
<td>0.064</td>
<td>0.626</td>
</tr>
<tr>
<td>Silt</td>
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<td>1330, 4670</td>
<td>0.081</td>
<td>0.011</td>
<td>0.886</td>
</tr>
<tr>
<td>Silty clay</td>
<td>1</td>
<td>1420</td>
<td>0.320</td>
<td>0.000</td>
<td>0.480</td>
</tr>
<tr>
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<td>1371, 1372</td>
<td>0.345</td>
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<td>0.547</td>
</tr>
<tr>
<td>Sand</td>
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<td>4650, 4651, 4660</td>
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<td>0.014</td>
<td>0.053</td>
</tr>
<tr>
<td>Sandy clay loam</td>
<td>5</td>
<td>1183, 1184, 2630, 4602, 4621</td>
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<td>0.039</td>
<td>0.270</td>
</tr>
<tr>
<td>Sandy loam</td>
<td>12</td>
<td>1381, 1390, 1391, 1392, 2530, 2560, 2583, 2751, 2753, 2762, 2764, 2765</td>
<td>0.096</td>
<td>0.057</td>
<td>0.303</td>
</tr>
</tbody>
</table>

* The mean value.

(Fig. 3). The DE was fitted to the selected WRC data using the nonlinear curve fitting routine in the Origin® 8.0 computer program (OriginLab Corporation, Northampton, MA, U.S.A.). In this case, C, A1, A2, h1 and h2 were all used as fitted parameters.

Dexter et al. (2008a) adopted the WRC range of soil suction 10 ≤ h ≤ 15,000 cm. We followed the lower limit and adopted 10 cm ≤ h ≤ 2000 cm in this study. Then, we assumed that the 10 cm suction on the WRC to be the boundary between the structural pores and macro pores. The porosity at 10 cm suction was expected to be the sum of parameters CA1, A2 in theory and defined as Φ1 (cm³ cm⁻3) in this study. The remaining porosity was defined as the macro porosity which may be produced by tillage or biological activity (Dexter and Richard, 2009a; Kutilek, 2004).

To quantitatively analyze the relationship between soil separates and pores, we unified parameters CA1, and A2 into non-dimensional parameters, and defined the following:

\[ S_1 = \frac{C}{\Phi_2} \] (5)

\[ S_m = \frac{A_1}{\Phi_2} \] (6)

\[ S_i = \frac{A_2}{\Phi_2} \] (7)

where \( \Phi_2 \) is the actual sum of parameters C, A1, and A2 (cm³ cm⁻³) in the fitting processes; and \( S_1, S_m \) and \( S_i \) are the residual, matrix, and structural void ratios, respectively. Thus, the three soil pore spaces were denoted with percentages, which were similar to those of the three soil separates. In this way, \( \Phi_1 \) could be used to obtain the suction boundary between the structural and macro pores by comparing it with \( \Phi_2 \). When \( S_1 \) was known, its corresponding suction head (defined as \( h_{\text{sat}} \), i.e., the suction boundary point between \( S_m \) and \( S_i \) ) could be obtained from the accumulative WRC (e.g., Fig. 2d). Furthermore, \( h_{\text{sat}} \) was used to calculate the pore radius \( r_{\text{pm}} \) according to Eq. (4). Finally, based on Eq. (3), the sand lower limit (i.e., the particle size boundary between sand and silt; \( d_s = 0.005 \text{ mm} \)) along with the pore radius \( r_{\text{pm}} \) could be used to calculate the value of \( \gamma \). The accuracy of the DE was quantified in terms of the root mean square errors (RMSE) between the measured and predicted water contents as follows:

\[ \text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - x_i)^2} \] (8)

where \( y_i \) is the predicted value, \( x_i \) is the measured value, and \( N \) is the sample number.

4. Results and discussion

4.1. Test of the DE

A good agreement between the measured and fitted water contents indicated that DE had desirable performance over a wide range of soil suctions (Fig. 4). The mean values of RMSE ranged from 0.004 cm³ cm⁻³ in clay and silty clay loam soils to 0.020 cm³ cm⁻³ in silt-textured soil (Table 2). The small mean values of RMSE showed that DE was applicable in a wide range of soil textures.
Additionally, if the measured soil total porosity (TP, cm$^3$ cm$^{-3}$) was greater than $\Phi_2$, an additional exponential term with the parameters $A_3$ (macro pore) and $h_3$ (suction at $A_3$) should be added to Eq. (2) (Dexter et al., 2008a). The inspection of $\Phi_1$, $\Phi_2$, and TP values (Table 2) showed that $\Phi_1 < \Phi_2 < TP$ in most cases, thereby suggested that using the range of $h < 10$ cm for macro pores was reasonable. However, most $\Phi_1$ values were smaller than $\Phi_2$ values. Therefore, the range of macro pore suctions should fall within $h < 10$ cm, which was similar to the result of Dexter et al. (2008a).

4.2. The three soil pore spaces linked to soil separates

The fitted parameters of the DE for a wide range of soil textures were summarized in Table 3. The largest value of parameter $C$ was 0.288, which appeared in the clay-textured soil, whereas the smallest value was 0.047, which appeared in the sand-textured soil. The mean value of parameter $A_2$ ranged from 0.079 for clay-textured soil to 0.346 for sandy soil. Parameter $C$ increased with increasing clay content, and the values of parameter $A_2$ generally increased with increasing sand content. Parameter $C$ and $A_2$ were associated with clay and sand contents, respectively. Meanwhile, the mean values of $A_1$ ranged from 0.050 in sandy soil to 0.174 on loam soil. It was found that the higher the silt contents were, the larger the $A_1$ values were for soils of silt loam, loam, and silt. $S_r$ increased with clay content (Fig. 5a) and decreased with sand content (Fig. 5c). $S_r$ and silt content showed no clear trend (Fig. 5b). The correlation between $S_r$ and clay content and the correlation between $S_r$ and sand content were fitted with the whole database respectively as follows:

$$S_r = 0.0921C_{clay} + 0.144, r^2 = 0.52$$

where $C_{clay}$ is the clay content, and $C_{sand}$ is the sand content. Interestingly, the relationships between $S_m$ and the three soil separates were apparently disordered. In contrast to $S_r$, $S_r$ generally decreased with increasing clay content (Fig. 6a) and increased with increasing sand content (Fig. 6c). The correlations between $S_r$ and clay content or sand content were fitted respectively by the

![Fig. 4. Measured soil water content from the selected samples of UNSODA vs. fitted water content by DE.](image-url)

### Table 2

<table>
<thead>
<tr>
<th>Texture</th>
<th>TP (cm$^3$ cm$^{-3}$)</th>
<th>$\Phi_1$ (cm$^3$ cm$^{-3}$)</th>
<th>$\Phi_2$ (cm$^3$ cm$^{-3}$)</th>
<th>RMSE (cm$^3$ cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>Clay</td>
<td>0.520</td>
<td>0.041</td>
<td>0.519</td>
<td>0.043</td>
</tr>
<tr>
<td>Silty clay</td>
<td>0.639</td>
<td>0.101</td>
<td>0.561</td>
<td>0.124</td>
</tr>
<tr>
<td>Silty clay loam</td>
<td>0.542</td>
<td>0.088</td>
<td>0.486</td>
<td>0.085</td>
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<tr>
<td>Clay loam</td>
<td>0.465</td>
<td>0.059</td>
<td>0.402</td>
<td>0.043</td>
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<tr>
<td>Sandy clay loam</td>
<td>0.457</td>
<td>0.007</td>
<td>0.420</td>
<td>0.020</td>
</tr>
<tr>
<td>Silt loam</td>
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<td>0.000</td>
<td>0.520</td>
<td>0.000</td>
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<tr>
<td>Loam</td>
<td>0.425</td>
<td>0.004</td>
<td>0.431</td>
<td>0.019</td>
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<tr>
<td>Silt</td>
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<td>0.038</td>
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<td>0.031</td>
<td>0.455</td>
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<tr>
<td>Sand</td>
<td>0.482</td>
<td>0.049</td>
<td>0.403</td>
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### Table 3

<table>
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<tr>
<th>Texture</th>
<th>$C$ (cm$^3$ cm$^{-3}$)</th>
<th>$A_1$ (cm$^3$ cm$^{-3}$)</th>
<th>$h_1$ (cm)</th>
<th>$A_2$ (cm$^3$ cm$^{-3}$)</th>
<th>$h_2$ (cm)</th>
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<td>SD</td>
<td>Mean</td>
<td>SD</td>
<td>Mean</td>
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<tr>
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<td>0.047</td>
<td>0.018</td>
<td>0.050</td>
<td>0.021</td>
<td>960.2</td>
</tr>
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</table>
Sand content had the greatest influence on structural pores. No structural pore space and only a mono-modal POD existed in the absence of sand (Dexter et al., 2008a). A linear function, e.g., Eq. (12), further explained how sand content influenced the structural porosity.

The synchronization in cumulative values clearly showed the symmetry between the soil particles and soil pores (Fig. 2). This observation provided a direct evidence that soil pore hierarchy could be associated with soil separates. In this study, residual pore space increased with clay content and the structural pore space increased with sand content as showed in Eq. (9) and Eq. (12), respectively. However, the correlation between matrix porosity and silt content was uncertain. Given that the sum of $S_r, S_m$, and $S_s$ is equal to 1, $S_m$ could be calculated with the following:

$$S_m = 1 - (S_r + S_s)$$  \hspace{1cm} (13)

When the soil texture was known, the linear functions that used $C_{clay}$ to predict $S_r$, $C_{sand}$ to predict $S_m$, and Eq. (13) could be regarded as simple empirical models to estimate soil pore spaces corresponding to the water retention characteristic.

A cross-validation method (Pachepsky and Rawls, 1999) was used in this study to quantify the accuracy of linear functions, e.g., Eq. (9), Eq. (12) and Eq. (13) in estimating the different soil porosities. The random splitting of data into the development and validation subsets was repeated 10 times and we used the ratio of 2:1 to split data into development and validation sets in each replication. Both RMSE and $R^2$ were calculated to enable
development and validation subsets to characterize the accuracy and the reliability of the functions, respectively.

As shown in Table 4, the highest accuracy appeared in linear functions that used $C_{\text{clay}}$ to predict $S_s$, where the RMSEs were 0.110 and 0.114 in the development and validation data sets, respectively, and the values of $R^2$ were 0.509 and 0.571 in the development and validation data sets, respectively. In contrast, the RMSEs of the linear functions that used $C_{\text{sand}}$ to predict $S_s$ were 0.139 and 0.144 in the development and validation data sets, respectively, and their $R^2$ values were 0.258 and 0.304 in the development and validation data sets, respectively. The prediction of $S_m$ may be the worst as Eq. (13) had the lowest values of $R^2$.

The low accuracy of linear functions predicting $S_m$ and $S_s$ may be attributed to the fact that only soil separates were considered when the relationships between soil texture and PODs were examined based on DE in this study. Soil POD is generally influenced by many other soil properties, such as organic carbon (Dexter et al., 2008b; Rawls et al., 2003), bulk density (Schaap and Leij, 1998; Schaan et al., 2001), and cation exchange capacity (Hodnett and Tomassella, 2002), which were not considered in this study. Moreover, the function using $C_{\text{sand}}$ to predict $S_s$ should include additional consideration of either natural or artificial soil compaction because it mainly influenced the macro or structural porosity of soils. Soil texture might be the key factor for the formation of soil pore structure among different factors in nature. However, only one key factor cannot effectively describe the pore structure of soils because of the inherent temporal and spatial variability of soil physiochemical properties. Therefore, a hierarchical approach (Schaap et al., 1998) may be helpful to explore multifactor effects on soil POD based on DE in the further study.

The comparison indicated that Eqs. (9), (12), and (13) performed well in predicting soil void ratios (Fig. 7). In practice, if $S_s$, $S_m$, or $S_l$ was possibly less than zero in calculation, the value could be set to zero. The new definitions of $C_l$, $A_l$, and $A_s$ by soil separates were reasonable, and the classification with soil separates made the hierarchy of soil pores distinct.

4.3. Correlation between DE differential functions and soil texture

In this study, the differential function of the WRC was directly regarded as the soil POD. According to the symmetry between soil POD and PSD, the soil POD based on DE (PODDE) can be analyzed with various soil types. The PODDE expression can be obtained as:

$$f(h) = \frac{dv}{d(\log h)} = \frac{A_1}{h_1^e} \left( \frac{h}{h_1} \right)^{h_1} - \frac{A_2}{h_2^e} \left( \frac{h}{h_2} \right)^{h_2}$$

PODDE curves generally had two cases: mono-peak, e.g., Fig. 8a–c, and the bi-peak, e.g., Fig. 8d–i. In bi-peak, the left peak was called the structural peak, which was associated with the structural pore space; the right peak was called matrix peak, which was associated with the matrix pore space (Fig. 8d). The peak value (PV), which reflects the height of the peak on PODDE curve, was introduced to define a quantitative criterion for PODDE curves, i.e., PV was calculated by substituting $h_1$ and $h_2$ into Eq. (14), where $PV_1$ at $h_1$ represented the matrix peak and $PV_2$ at $h_2$ represented the structural peak. Thus, if $PV_1 = PV_2$ and $h_1 = h_2$, the PODDE curve would be mono-peak; if the $PV_1 \neq PV_2$ or $h_1 \neq h_2$, the PODDE curve would be bi-peak.

Furthermore, the occurrence of the bi-peak could be divided into two situations: high structural peak-low matrix peak (HL; Fig. 8d–f) and low structural peak-high matrix peak (LH; Fig. 8g–i). For subdivision, we decided on $|PV_1 - PV_2| = 0.1$ as the limit based on our data set, i.e., further division existed in the HL based on whether or not $|PV_1 - PV_2| > 0.1$. HL was further divided into high structural peak-low matrix peak (H–L), where $|PV_1 - PV_2| \leq 0.1$, and high structural peak-extremely low matrix peak (H–EL), where $|PV_1 - PV_2| > 0.1$. Similarly, LH was also divided into low structural peak-high matrix peak (L–H) and extremely low structural peak-high matrix peak (EL–H).

A few representative situations from the preceding cases were initially analyzed to ensure the efficacy of this procedure in a more general sense. Mono-peak, H–EL and EL–H could be considered as the representative scenarios of PODDE because of their typical peak, and therefore were analyzed first. Our next step would be to explore the relationship between the representative scenarios of PODDE and soil texture, and then apply the relationship to general situations.

4.3.1. Representative scenarios

In the mono-peak scenarios listed in Table 5, soil texture was one dominant component system, where silt content was greater than 0.8. In this situation, the matrix porosity ($A_1$) and structural porosity ($A_2$), as well as the values of $h_1$ and $h_2$ values, were nearly equal. This phenomenon indicated that the structural and matrix pore spaces could be merged into one pore system dependent on Eq. (2). Thus, the following equation could be obtained:

$$w = C + Ae^{(-h/h_0)}$$

where $A = A_1 + A_2$ and $h_0 = h_1 = h_2$. The differential function of Eq. (15) could be obtained as follows:

$$g(h) = \frac{dv}{d(\log h)} = \frac{A_1}{h_0^e} \left( \frac{h}{h_0} \right)^{h_0}$$

Eq. (16) manifests only one PV. Therefore, the soil PODDE in this case would show as mono-modality (e.g., Fig. 8a–c). Eq. (15) indicated that most of the pores might be located around the silt fraction. Thus, the pores were shown as one dominant pore system. When one soil separate was dominant, its associated pores would have a dominant fraction as well.

In the first five soil samples of the bi-peak H–EL scenario (Table 5), sand was the dominant soil separate, where sand content was greater than 0.64. The structural porosity correspondingly occupied the majority in $Q_2$, where $S_s$ was greater than 0.55. Although a bi-peak was present, one dominant soil separate led to a dominant pore system (e.g., Fig. 8a and f). Moreover, the values of parameters $C A_1$, and $A_2$ were proportional to the clay, silt, and sand contents, respectively, in the five codes. Silt was the dominant soil separate in the latter three soil samples of the H–EL scenario (Table 5). A greater silt fraction should correspond to greater matrix porosities. However, the structural pores instead of matrix pores occupied the majority in $Q_2$. In the bi-peak EL–H scenario, silt was the dominant soil separate, and matrix porosity occupied the majority in $Q_2$ except for sample code 2603, which indicated that the dominant soil separate was consistent with the dominant soil pore space (e.g., Fig. 8h and i).

Therefore, one dominant soil separate would result in one dominant soil pore space regardless of whether PODDE was mono- or bi-modal as caused by the soil separates. In particular, if one soil
separate occupied the absolute majority, the PODDE would tend to generate mono-peaks, whereas the others would tend to generate bi-peaks.

4.3.2. General situations

From the selected 78 soil samples, 75 samples were available for the analysis of PODDE with soil texture, which included 33 samples of HL, 38 samples of LH, and 4 samples of mono-peak. In most of HL samples, $S_s$ was greater than $S_m$ or $S_v$, whereas in all of the LH samples, $S_s$ was the smallest. In general, if the $S_v$ was greater than $S_m$, PODDE was HL; otherwise, it was LH.

Both triangles of soil texture and pores were shown in Fig. 9 to further explore the relationship between their corresponding soil texture in the triangle (Fig. 9a). Moreover, 18 samples showed inconsistency between soil texture and void ratios in the triangle (Fig. 9b) in the 33HL samples. Similarly, among the samples of LH, 15 samples showed consistent distributions between their soil texture and void ratios in the triangle (Fig. 9c), whereas 23 samples showed inconsistencies (Fig. 9d). However, the silt content was greater than 0.4 in most samples shown in Fig. 9b and d, and most of the soil void ratios were located in the central area of the triangles.

The clay content was less than 0.3 in most of the HL samples (Fig. 9a and b). This finding indicated that the structural peak was slightly influenced by clay separates, and was mainly influenced by silt and sand separates. The sand content in most of the 38 samples of LH was less than 0.5 (Fig. 9c and d). When the sand content was greater than 0.5, the PODDE curve would tend to be HL (Fig. 9). Furthermore, all PODDE curves of the clay-textured soils were shown as LH. All PODDE curves of sand and 10 sandy loam-textured soils were shown as HL. The matrix peak was greatly influenced by clay content, and the structural peak was greatly affected by sand content.

The determining criteria of soil separates (i.e., the particle sizes of clay, silt, and sand) were empirical and did not adequately consider the size relationship between the soil particles and pores. In this study, the soil packing-related parameter $\gamma$ was used to explain the uncertain relationship between soil texture and pores in Fig. 9b and d. We used UNSODA code 1281 in Table 5 as an example for dominant silt content. As shown in Fig. 2, different values of $\gamma$ resulted in different fractions of the three soil void ratios. When $\gamma = 1$ and $\gamma = 10$, $S_m$ was the highest (Fig. 2a and b). However, when $\gamma = 20$, $S_v$ became the highest (Fig. 2c). In reality, $\gamma$ was equal to 62.6, and $S_r$ had the highest value (Fig. 2d). The soil texture determined the form of soil PSDs, but the parameter $\gamma$ determined the location in the figures (e.g., Fig. 2). Thus, $S_m$ or $S_v$ might be the dominant soil void ratio when the dominant soil separate was silt. Then, LH or HL PODDE might appear.

The parameters of $h_1$ and $h_2$ were actually the characteristic suction values, which were used to calculate the PV of PODDE. Rather than the size boundaries of soil pores, parameters $h_1$ and $h_2$ were the most representative values where the matrix and structural pore suction values were concentrated, respectively. If we regarded the radius $r_1 = 0.149/h_1$ and $r_2 = 0.149/h_2$ as the average matrix pore and structure pore radii, respectively, then we could determine how the average matrix and structural pore radii change in association with the changes of $h_1$ and $h_2$ values. In practice, this method could help reveal how tillage practices or soil amendments improve soil pore structure.

5. Implication of the current study

DE can provide soil pore structure information, which may be useful for interpreting and predicting of other soil properties. Large PODDE values reflect fast water desorption rates, which is indicated by the greatest WRC slope corresponding to the PV, as shown in Fig. 10. Additionally, large PODDE values correspond to great volumes of soil pores. The volume of corresponding pore space fast increased when pore suction approaches $h_1$ or $h_2$ on the PODDE. Then a small change in pore suction results in a large change in pore space (i.e., water content). Therefore, the water desorption rate in this case would change immediately (i.e., large WRC slope). In this way, the water desorption rate is strongly associated with POD as well as PSD, according to Eqs. (3) and (4). Therefore, DE provides a connection among POD, PSD, and water desorption rate, and can be used to further explain the changes in soil moisture dynamics arising from soil texture.

The applicability and accuracy of regression models are limited mainly by unsuccessful extrapolation of regression equations beyond their development region (Hwang and Choi, 2006). As an empirical model, DE has some limitations such as the extrapolation to dry and saturated portions. As shown in Fig. 10, a WRC predicted by the DE quickly became parallel to the x-axis near the last measured point. Dexter et al. (2012) stated that this phenomenon was related to hydraulic cutoff. Hydraulic cutoff describes the point at which the soil residual water remains in the soil after the connected (drainable) textural pore space is emptied by Darcian convective flow, where water moves considerably slow and mainly through vapor-phase diffusion. Extrapolation to the large suction head can be solved with DE together with the Groenevelt and Grant (2004) equation. However, obtaining a complete WRC in practice through this method is time consuming and tedious. Moreover, the
WRC data from near saturation are often lost. Extrapolation to saturation, which is regarded as the macro pores in DE, has been minimally investigated. This problem may be resolved by a trinodal soil water retention equation (Dexter and Richard, 2009a). However, numerous verifications are needed.

Some important issues could not be addressed in our investigation. Parameter $\gamma$ greatly influenced the forms of the PODDE curve. Its value is related to the packing pattern of soil particles. This finding should be confirmed through other analyses, e.g., the technologies of X-ray (Beckers et al., 2014; Munkholm et al., 2012) and original SEM (Dathe et al., 2001). We believe that the proposed structural models cannot be confirmed without these analyses. In addition, the main drawback of the continuous form-based models of water retention, e.g., van Genuchten model and DE, is their inability to describe air entry pressure or the discontinuity of WRCs near the saturation point. The soil pores corresponding to air entry suction on the WRCs may be located in a macro pore space. The discontinuity of the WRCs near the
Table 5
The detailed information of the representative cases of PODH, including mono-peak, H-EL and EL-H.

<table>
<thead>
<tr>
<th>Cases</th>
<th>UNSODA Code</th>
<th>Clay</th>
<th>Silt</th>
<th>Sand</th>
<th>C (cm³ cm⁻¹)</th>
<th>A₁ (cm³ cm⁻¹)</th>
<th>h₁ (cm)</th>
<th>A₂ (cm³ cm⁻¹)</th>
<th>h₂ (cm)</th>
<th>PV₁</th>
<th>PV₂</th>
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<td>0.152</td>
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<td>0.900</td>
<td>0.030</td>
<td>0.103</td>
<td>0.161</td>
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<td>152.8</td>
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</table>

Fig. 9. Soil pore distribution (i.e., residual, matrix, and structural pores) in the soil pore triangle vs. the soil textural composition in the soil texture triangle. Samples in (a) and (b) belong to HL PODH curves; the samples in (c) and (d) belong to LH PODH curves; the soil samples in (a) and (c) followed their void ratio distribution consistently with their corresponding soil textural computation in the triangles; the soil samples in (b) and (d) showed inconsistency between soil texture and void ratios.
saturation point may be resolved by piecewise functions of a double-exponential equation, or tri-exponential equation (Dexter and Richard, 2008a; Dexter et al., 2008a), where the air entry suction needs further determination in a macro pore space. This topic is an interesting research field in future studies.

6. Conclusion

In this study, we established some connections between the three soil separates (i.e., clay, silt, and sand) and the three terms of the DE. Moreover, the mono- and bi-modality of DE differential functions were analyzed with soil texture. The symmetry between POD and PSD could be used to quantitatively analyze the different soil pore spaces and the causes of the mono- or bi-modality of DE differential functions. Our conclusions are as follows:

1. DE performed reasonably well with a wide range of soil texture. DE had suitable parameters, accessibility, and reasonably separate physical meanings. Therefore, it provides a simple and practical tool for WRC prediction.

2. Clay and sand content could be fitted with residual and structural pores, respectively, based on DE by linear functions, which could be used to predict soil porosities with soil texture.

3. The mono- or bi-modality of DE differential is related to the soil separates. One dominant soil separate would result in one dominant soil pore space. If this soil separate occupied the absolute majority, the PODDE would tend to generate mono-peaks, whereas the others would tend to be bi-peaked.

4. In general, if $S_s$ was greater than $S_m$, PODDE would be HL; otherwise, it would be LH. Furthermore, the matrix peak was greatly influenced by clay content, and the structural peak was greatly affected by sand content. When the sand content was greater than 0.5, the PODDE curve would tend to be HL.

Acknowledgements

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