Transport Capacity of Overland Flow with High Sediment Concentration

Bofu Yu1; Guang-hui Zhang2; and Xudong Fu3

Abstract: The concept and estimation of sediment transport capacity of overland flows are pivotal to soil erosion, sediment transport, and deposition modeling. There is a limited understanding of the effect of high sediment concentration on the transport capacity of overland flow, although sediments in suspension are known to affect turbulent mixing and settling velocity in rivers. A new functional relationship between a dimensionless parameter involving stream power and settling velocity and the volumetric concentration at the transport limit was developed using a set of flume experiments with slope up to 46.6%, unit discharge up to 50 cm² · s⁻¹, median particle size of 0.326 mm, and sediment concentration up to 1,140 kg · m⁻³. The new relationship has two theoretical limits on sediment concentration at the transport limit. Under low flow conditions, the sediment concentration is limited by the available stream power. At high stream power, the sediment concentration is limited by the space available in flow to accommodate sediments in motion. As a predictor of the sediment concentration at the transport limit, the new relationship worked very well with the Nash-Sutcliffe coefficient of efficiency of 0.95 and was shown to be superior to empirical relationships based on stream power and other commonly used predictors of the transport capacity for rivers. The paper also shows that formulas for the transport capacity which have been validated and widely used for rivers with high sediment concentrations are inaccurate and should not be used to predict the transport capacity of overland flow. DOI: 10.1061/ASCE.HE.1943-5584.0000998. © 2014 American Society of Civil Engineers.

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Introduction

The sediment transport capacity of overland flow defines the maximum amount of sediments the flow can carry given the flow rate, topography, surface characteristics, size, and density of sediments. The transport capacity is often expressed in terms of sediment discharge per flow width, 

\[ T_c (kg \cdot s^{-1} \cdot m^{-1}) \]

or more fundamentally, in terms of the corresponding sediment concentration at the transport limit, 

\[ c_s (kg \cdot m^{-3}) \]

(Foster and Meyer 1972; Hairsine and Rose 1992a; Yu 2003). The concept of sediment transport capacity plays a pivotal role in soil erosion research, for the transport capacity defines areas of net erosion where the actual sediment discharge falls below the transport capacity and that of net deposition where the transport capacity is exceeded. Most of the process-based erosion prediction models include a module to evaluate the magnitude of the sediment transport capacity for a given set of hydraulic variables, sediment characteristics, and required model parameter values.

Water erosion prediction project (WEPP), for instance, uses Yalin’s bed load formula to evaluate the sediment transport capacity (Foster 1982; Finkner et al. 1989; Nearing et al. 1989; Laflen et al. 1997; Flanagan and Nearing 2000). Yalin was originally motivated to improve Einstein’s formula for bed load transport when the rate of sediment discharge is high (Yalin 1963). The data on which this improvement was based were collected using flume experiments with coal, barite, and gravel of uniform size ranging from 0.315 to 28.6 mm (Einstein 1942). While material of this caliber is not typical of sediments carried by overland flows, Yalin’s formula is implemented in WEPP because the formula performed favorably in comparison with several other formulas (Alonso et al. 1981). For Griffith University Erosion System Template (QUEST), the sediment transport capacity is seen as a result of dynamic balance of simultaneous erosion and deposition processes (Hairsine and Rose 1992a; Misra and Rose 1996; Rose et al. 2011). The conceptual basis for this approach can be traced back to Bagnold who assumed that a fraction of stream power would be expended on sediment transport (Bagnold 1966; Rose et al. 1983a, b). Experiments were carried out to evaluate this fraction with fine sediments and low flow rates on fairly gentle slopes (Proffitt et al. 1993). For the European Soil Erosion Model (EUROSEM) (Morgan et al. 1998), Govers’ empirical equations using the unit stream power are implemented (Govers 1990, 1992). The unit stream power is used in the context of sediment transport prediction by Yang (1972, 1973). The parameter values for these empirical relationships could be determined from the median particle size (Govers 1990; Morgan et al. 1998). Experiments were conducted to closely represent overland flow conditions with slope varying from 1 to 12° (1.75–21.3%) and unit discharge varying from 2 to 100 cm² · s⁻¹ (Govers 1990). Assuming sheet flow and a slope length of 20 m, this unit discharge would imply an equivalent rainfall intensity of 36–1,800 mm · h⁻¹. The median particle size varied from 0.058 to 1.098 mm (Govers 1990).

In addition to these empirical or semiempirical approaches to prediction of the sediment transport capacity, a number of experiments were carried out to determine the sediment transport capacity...
of overland flows. These experiments were not particularly designed to test sediment transport formulas; instead, the results are mostly used to test which hydraulic parameters are best correlated with the observed transport capacity. Zhang et al. (2009) measured the transport capacity of flows ranging from 6.25 to 50 cm² · s⁻¹. The slope varied from 8.8 to 46.6% and Zhang et al. (2009) found that the stream power is the best predictor of the transport capacity in comparison with the shear stress and unit stream power. Wang et al. (2012) ran a similar set of experiments (unit discharge of 2.2–6.7 cm² · s⁻¹; slope of 15.8–38.4%) and developed regression equations for the transport capacity using the unit discharge and slope as independent variables. Experiments by Ali et al. (2012) with unit discharge of 0.7–20.7 cm² · s⁻¹ and slopes of 5.2–17.6% show that the unit stream power is a better predictor of the transport capacity. Ali et al. (2013) subsequently compared a number of commonly used predictors and proposed yet another equation that better fits the data from their flume experiments.

With steep slopes and high unit discharge for overland flow experiments, the measured sediment concentration for these experiments can be very high. For example, the volumetric concentration reached 0.27–0.28 for the two fine materials used (Govers 1990). Sediment concentration as high as 1,254 kg · m⁻³ was measured (Zhang et al. 2009) and about 570 kg · m⁻³ with a much lower flow discharge (Wang et al. 2012). Sediment concentration reached 320 kg · m⁻³ even on gentle slopes of up to 17.6% (Ali et al. 2012). In nature, sediment concentration of overland flows could be quite high as well. There are well documented case studies of hyper-sediment concentration of overland flows and of runoff from small watersheds on the Loess Plateau in excess of 2000 kg · m⁻³. Hessel (2006) reviews a number of case studies where high sediment concentrations well in excess of 1,000 kg · m⁻³ have been reported. In general, examinations of the effect of high concentration on the transport capacity of sediment-laden flows are fairly limited for overland flows.

In contrast, the effect of high sediment concentration on river flows has been much researched. Flows with sediment concentration in excess of 200–300 kg · m⁻³ are thought to be hyperconcentrated flows (Wan and Wang 1994; Xu 1999; van Maren et al. 2009). Existence of sediments in suspension, especially fine sediments, is thought to affect the density of sediment-water mixture, turbulence dynamics, and the settling velocity (Wan and Wang 1994). Much of the research has been used to develop sediment transport formulas to model scouring and deposition along river channels. Of note, the formula proposed by Zhang and Zhang (1992) has been extensively tested and implemented to simulate sediment transport processes in the lower Yellow River where hyperconcentrated flows with concentration in excess of 200–300 kg · m⁻³ have occurred 40 times between 1960 and 1992 (Shu 1993; Zhang et al. 2006; van Maren et al. 2009). This formula has taken into consideration the effect of sediment concentration on the density of sediment-water mixture, van Kármán’s constant, and the settling velocity (Zhang and Zhang 1992).

The objectives of this paper were to evaluate the effect of sediment concentration on the transport capacity of overland flows and to evaluate well established formulas developed for river flows with high sediment concentration.

### Data and Method

A set of 320 measurements of the sediment concentration at the transport limit were taken using eight distinct flow rates (0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, and 2 L · s⁻¹), and 8 different slopes (5, 10, 12.5, 15, 17.5, 20, 22.5, and 25°) (Zhang et al. 2009). The flume was 0.38-m wide and 5-m long. Test sediment, mostly alluvial sand, was sourced from the Yongding River near Beijing. Density of the sediment was 2,400 kg · m⁻³, and d₁₀, d₅₀, and d₉₀ of the sediments were 0.134, 0.326, and 0.608 mm, respectively. The flume bed was fixed and glued with the test sediment to maintain similar grain roughness for all experiment runs. For each slope-flow rate combination and before sediment was introduced, water depth was measured with a level probe at 0.6 m upstream from the end of the flume. Twelve such depth measurements were taken, and the maximum and minimum water depths were then eliminated. The mean water depth was the average of the remaining 10 measurements. Flow velocity was measured using a fluorescent dye technique, and the leading edge velocity was multiplied by a factor of 0.8 to obtain the mean velocity (Luk and Merz 1992). Sediment was fed using a 1 m³ hopper over the flume, and the feeding rate was controlled with a rotor inside the hopper. For each slope-flow rate combination, the feeding rate was gradually increased until the fed sediment could not be carried down the flume, and some deposition started to occur. In addition, a 20 cm slot filled with the test sediment was placed 0.5 m from the end of the flume as the second source of sediment to ensure the sediment transport limit is reached. The slot was covered with a thin iron sheet initially, and this cover was not removed until the maximum feeding rate had been reached for each slope-flow rate combination. For each slope-flow rate combination, five replicates of the sediment concentration were undertaken. The average of the five concentrations was assumed to be the sediment concentration at the transport limit for that slope-flow rate combination. Measured mean water depth ranged from 0.9 to 5.7 mm (Zhang et al. 2009). Measured sediment concentration at the transport limit ranged from 84.7 to 1,254 kg · m⁻³ among the 320 individual measurements. The effect of sediment concentration on flow hydraulics for a range of flow conditions has been reported elsewhere (Zhang et al. 2010a, b).

For this paper, measured mean water depth, mean velocity, and sediment concentration at the transport limit for three selected runs are presented in Table 1 to indicate the range of flow conditions and observed hydraulic variables and sediment concentrations at the transport limit.

For this set of experiments, samples of water-sediment mixture were collected. The total volume of the sample and dry weight of sediments were subsequently measured. Given that Vₜ and Vₜ are the volumes of sediment and water, respectively, and Mₛ is the mass of sediments, the sediment concentration at the transport limit, cₜ, in kg · m⁻³ was computed as

\[
cₜ = \frac{Mₛ}{Vₛ + Vₜ}
\]

### Table 1. Measured Hydraulic Variables and Sediment Concentration at the Transport Limit for Three Selected Slope-Flow Rate Combinations

<table>
<thead>
<tr>
<th>Experimental setup</th>
<th>Run 1</th>
<th>Run 28</th>
<th>Run 64</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope (degree)</td>
<td>5</td>
<td>15</td>
<td>25</td>
</tr>
<tr>
<td>Flow rate (L · s⁻¹)</td>
<td>0.25</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td>Mean water depth (mm)</td>
<td>1.4</td>
<td>2.7</td>
<td>3.0</td>
</tr>
<tr>
<td>Mean velocity (m · s⁻¹)</td>
<td>0.44</td>
<td>0.94</td>
<td>1.67</td>
</tr>
<tr>
<td>Concentration (kg · m⁻³)</td>
<td>102</td>
<td>617</td>
<td>1,124</td>
</tr>
</tbody>
</table>
The sediment discharge, $G$ in kg · s$^{-1}$, is by definition given by

$$G = \frac{M_g}{V_w/Q}$$

where $Q =$ discharge set and controlled at the upper end of the flume. The equation above can be rearranged in terms of $c_i$ and the volumetric sediment concentration, $c_v$, as follows:

$$G = \frac{c_i Q}{1 - c_v}$$  \hspace{1cm} (2)

where the concentration by volume, which is dimensionless, is given by

$$c_v = \frac{c_i}{\rho_s}$$  \hspace{1cm} (3)

and $\rho_s =$ sediment density. In literature, this volumetric concentration is also referred to as the relative concentration used to define hyperconcentrated flows (Costa 1988; Bradley and McCutcheon 1987). Unit discharge, $q$, and unit sediment discharge, $q_s$, were determined by dividing $G$ and $Q$ with the width of flume, respectively.

Eqs. (1), (3), and (2) above were used to compute the sediment concentration at the transport limit, the corresponding volumetric concentration, and sediment discharge, respectively. It is worth noting that in Zhang et al. (2009), the unit sediment discharge was determined from the dry weight of sediments, average time taken to collect the sample, and the flume width. The method used in Zhang et al. (2009) was not as accurate as that outlined above because of the error and uncertainty involved in measuring the amount of time taken to collect the sample.

Given particle size, $d$, and density, Cheng’s method was used to compute the corresponding settling velocity (Cheng 1997). A dimensionless particle parameter, $d^*$, is defined as

$$d^* = \left[\frac{(\rho_s - \rho)g}{\rho \nu^2}\right]^{1/3} d$$  \hspace{1cm} (4)

where $\rho =$ water density (kg · m$^{-3}$), $g =$ acceleration due to gravity (9.81 m · s$^{-2}$), and $\nu =$ kinematic viscosity (m$^2$ · s$^{-1}$). Cheng (1997) shows that the settling velocity can be expressed as an explicit function of $d^*$ as follows:

$$\frac{\omega d}{\nu} = \left[\sqrt{25 + 1.24d^*_i} - 5\right]^{1.5}$$  \hspace{1cm} (5)

The method was tested and validated for particle size ranging from 10$^{-5}$ to 4.5 mm (Cheng 1997). Ten size classes were used to represent the particle size distribution. Particle size for $d_1$, $d_{15}$, . . . , $d_{95}$ and the corresponding settling velocities were assumed to represent these 10 classes as shown in Table 2. The average settling velocity (0.0354 m · s$^{-1}$) was taken as the arithmetic average of the 10 representative settling velocity values. Ten size classes are commonly used to represent the range of settling velocity characteristics (e.g., Beuselinck et al. 2002). Further investigation showed that doubling the number of classes to 20, the change in the average settling velocity was <0.1%. This is, however, an appreciable difference between the average settling velocity and that based on $d_{90}$, and the difference was >6% for the sediments considered in this paper.

Stream power, $\Omega$, in W · m$^{-2}$ was computed as

$$\Omega = \rho g q S$$  \hspace{1cm} (6)

where $q =$ discharge per flow width (m$^2$ · s$^{-1}$), and $S =$ slope (tangent of the slope angle). For flows with high sediment concentrations, water density in Eq. (6) should be replaced with the density of the water-sediment mixture given later in the paper.

A number of frameworks were considered in assessing the effect of sediment concentration on transport capacity. In the absence of rainfall, the governing equation for sediment movement developed for GUEST (Hairseine and Rose 1992a) can be written as

$$\frac{\partial(c_i D)}{\partial t} + \frac{\partial(c_i q)}{\partial x} = r_i + r_{ri} - d_i$$  \hspace{1cm} (7)

where $D =$ water depth (m), $c_i =$ sediment concentration for particle size class $i$ (kg · m$^{-3}$), $r_i$ and $r_{ri}$ = rates of flow entrainment and reentrainment (kg · m$^{-2}$ · s$^{-1}$), $d_i =$ rate of deposition (kg · m$^{-2}$ · s$^{-1}$), $x =$ distance in the downslope direction (m), and $t =$ time (s). Flow entrainment is used to describe the removal of sediment from the original cohesive soil mass, whereas reentrainment refers to the removal of deposited sediment of negligible cohesion on soil surface during an erosion event (Hairseine and Rose 1992a). Eq. (7) is based on mass balance for individual particle size classes. The three terms on the right hand side of Eq. (7) are modeled as follows:

$$r_i = (1 - H) \frac{F(\Omega - \Omega_0)}{NJ}$$  \hspace{1cm} (8)

$$r_{ri} = H \frac{\alpha_i \omega_i c_i}{(\rho_s - \rho) g D} M_{di} M_{dt}$$  \hspace{1cm} (9)

$$d_i = \alpha_i \omega_i c_i$$  \hspace{1cm} (10)

where $H$ is originally interpreted as the fraction of the original soil covered with deposited sediments (Hairseine and Rose 1992a), $M_{di}$ is the amount of sediment in size class $i$ in the deposited layer, and $M_{dt}$ is the total amount of deposited sediments. Alternatively, $H$ can be interpreted as the ratio of actual concentration over the concentration at the transport limit, i.e., $H = c/c_t$ (Yu 2003). The ratio $M_{di}/M_{dt}$ represents the fraction of sediment in size class $i$ in the deposited layer. Other variables in Eqs. (8)–(10) are defined as follows:

- $F$ is the fraction of stream power effective in entrainment and reentrainment;
- $N$ the number of size classes of equal weight;
- $J$ the specific energy of entrainment (J · kg$^{-1}$);
- $\Omega_0$, threshold stream power per unit area (W · m$^{-2}$);
- $\alpha_i$ is the ratio of the sediment concentration near the bed to the mean sediment concentration across the entire depth for class $i$ (Croley 1982); and
- $\omega_i$ is the settling velocity for size class $i$ (m · s$^{-1}$).

Table 2. Particle Size Distribution of the Sediments Used in this Experimental Study

<table>
<thead>
<tr>
<th>Size class</th>
<th>Size (mm)</th>
<th>Settling velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.092</td>
<td>0.004429</td>
</tr>
<tr>
<td>2</td>
<td>0.166</td>
<td>0.012582</td>
</tr>
<tr>
<td>3</td>
<td>0.217</td>
<td>0.018997</td>
</tr>
<tr>
<td>4</td>
<td>0.260</td>
<td>0.024754</td>
</tr>
<tr>
<td>5</td>
<td>0.303</td>
<td>0.030410</td>
</tr>
<tr>
<td>6</td>
<td>0.349</td>
<td>0.036282</td>
</tr>
<tr>
<td>7</td>
<td>0.401</td>
<td>0.042694</td>
</tr>
<tr>
<td>8</td>
<td>0.463</td>
<td>0.050103</td>
</tr>
<tr>
<td>9</td>
<td>0.546</td>
<td>0.059338</td>
</tr>
<tr>
<td>10</td>
<td>0.697</td>
<td>0.074801</td>
</tr>
</tbody>
</table>

Note: Settling velocity was estimated using Cheng (1997) and an assumed kinetic viscosity of $10^{-6}$ m$^2$ · s$^{-1}$.
For flow in rills, Hairsine and Rose (1992b) consider how energy expenditure is distributed between the rill bed and rill walls. The wall effect is not considered in this paper as the water depth of a few millimeters is about two orders of magnitude less than the width of the flume (380 mm).

Under the equilibrium conditions and when the sediment concentration reaches that at the transport limit, the $\partial/\partial x$ and $\partial/\partial t$ terms vanish, and $c = c_t$, or $H = 1$; the flow entrainment term also equals zero according to Eq. (8) and thus results in the following equation:

$$r_{ri} = d_i$$ (11)

Eq. (11) above indicates a dynamic balance between the rate of reentrainment and that of deposition for individual size classes when sediment concentration reaches its maximum. Summing the equations above over $N$ size classes leads to an analytical expression for the sediment concentration at the transport limit

$$c_t = \frac{F\rho_s(\Omega - \Omega_a)}{(\rho_s - \rho)gD\omega_a}$$ (12)

where $\omega_a$ = average settling velocity. The $M_{di}/M_{dt}$ terms sum to unity under equilibrium conditions. If a new dimensionless parameter, $\Lambda$, is defined as

$$\Lambda = \frac{\Omega}{(\rho_s - \rho)gD\omega_a}$$ (13)

then the following dimensionless relationship for the sediment concentration at the transport limit based on Eq. (12) is

$$c_t = F(\Lambda - \Lambda_a)$$ (14)

where $c_t$ = volumetric sediment concentration defined by Eq. (3). The parameter $F$ in the equation above is the fractional stream power involved in returning deposited sediments back in suspension or in motion. The parameter $\Lambda_a$ is related to the threshold stream power. In a well-designed experiment, Proffitt et al. (1993) show that the value for $F$ is approximately constant at $0.1 \pm 0.04$ and $0.14 \pm 0.05$ for the two soils tested.

In addition to this theoretical consideration of the dynamic balance between entrainment and deposition, an empirical formula of the form

$$T_c = a(\Omega - \Omega_a)$$ (15)

was considered for comparison purposes, where $T_c$ is the transport capacity in kg · m$^{-3}$ · s$^{-1}$. This measure of sediment transport capacity is related to $c_t$ and the unit discharge, $q$, via Eq. (2). Zhang et al. (2009) find that a linear relationship between the transport capacity, $T_c$, and stream power [Eq. (15)] is the best empirical relationship out of five different empirical formulations for the transport capacity for this set of experiments.

The authors have also considered the sediment concentration at the transport limit of the generic form

$$c_t = K \left( \frac{V^3}{gD\omega_a} \right)^m$$ (16)

where $K$ and $m$ are parameters to be estimated empirically. Eq. (16) involves dimensionless quantities and is based on the energy required to maintain sediments in suspension and is by far the most widely used formulation to predict sediment transport capacity in China (Zhang and Xie 1993). When using Eq. (16), the settling velocity needs to be adjusted to take into account the effect of sediment concentration on settling velocity. This is described in subsequent paragraphs.

Finally, a formula was tested for the sediment concentration at the transport limit developed by Zhang and Zhang (1992), which has been widely validated for both normal and hyperconcentrated flows in the Yellow River (Shu 1993; Zhang et al. 2006). In addition, this formula has been widely implemented in mathematical models to simulate unsteady sediment transport in the Yellow River (Zhang et al. 2002a, b; He et al. 2012). The formula is given as follows:

$$c_t = 2.5 \left[ \frac{(0.0022 + c_v)V^3}{\kappa \rho_s \rho_o gD\omega_a} \right] \ln \left( \frac{D}{6d_{50}} \right)^{0.62}$$ (17)

where $V$ = stream velocity (m · s$^{-1}$), $\kappa(\cdot)$ = von Kármán’s constant, and $d_{50}$ = median particle size of the bed material (m). Settling velocity, van Kármán’s constant, and density of the water-sediment mixture are thought to be affected by the sediment concentration. The density of sediment-laden flow or the effective density is related to sediment concentration, water, and sediment density as follows:

$$\rho_m = \rho + c_t \left( \frac{\rho_s - \rho}{\rho_s} \right)$$ (18)

The effect of the sediment concentration on the von Kármán’s constant is given by

$$\frac{\kappa}{\kappa_0} = 1 - 4.2 \sqrt{c_v} (0.365 - c_v)$$ (19)

where $\kappa_0$ = van Kármán’s constant for clear water, for which 0.4 is normally used. Settling velocity is generally reduced because of the high sediment concentration. The formula to quantify this effect is given by

$$\frac{\omega_s}{\omega_{so}} = \left( 1 - \frac{c_s}{2.25d_{50}} \right)^{3.5} (1 - 1.25c_v)$$ (20)

where $\omega_s$ = clear water settling velocity using Eq. (5) in this paper (Zhang and Zhang 1992). The $d_{50}$ in Eq. (20) refers to the median particle size of the suspended sediments in mm (Sha 1965; Zhang and Zhang 1992). For this paper, the authors did not differentiate between suspended sediments and bed-material, and $d_{50}$ of 0.326 mm was used for both Eqs. (17) and (20).

Results

**Relationship between Sediment Concentration at the Transport Limit and the Dimensionless Stream Power**

Fig. 1 shows the observed sediment concentrations at the transport limit as a function of the product of $\rho_s$ and the dimensionless parameter $\Lambda$. Water density was used to compute the stream power using Eq. (6). The slope of the relationship aligns with the parameter $F$ as in Eqs. (12) and (14). The error bars in Fig. 1 are based on variations in the measured sediment concentrations among the five replicates. It is shown that the variability among the five replicates is relatively small for this set of experiments. It is also shown that there is no fixed value for $F$ for this set of experiments involving steep slopes and high sediment concentrations. Eq. (14) with constant parameter values for $F$ and $\Lambda_a$ is therefore inappropriate to describe the relationship between the stream power and the sediment concentration at the transport limit. By fitting a straight line for observations when the sediment concentrations are...
<730 kg·m\(^{-3}\), an \(F\) value of 0.056 ± 0.002 with \(R^2 = 0.98\) (Fig. 1) is obtained forcing the line of best fit to go through the origin. On closer inspection of Fig. 1, for even smaller concentrations and smaller stream power, an \(F\) value closer to 0.1 and a threshold stream power may be obtained (results not shown). In general, however, Fig. 1 strongly indicates a nonlinear relationship between the stream power and the sediment concentration at the transport limit for this set of experiments.

To further explore the relationship between stream power and sediment concentration and taking into consideration the effect of sediment concentration on the density of sediment-laden flows, volumetric concentration is plotted against the dimensionless parameter \(\Lambda\) to render both axes dimensionless (Fig. 2). The density of water-sediment mixture, i.e., \(\rho_m\) as defined by Eq. (18), was used to compute the stream power [Eq. (6)] and the parameter \(\Lambda\) [Eq. (13)]. The scatter plot suggests a limiting behavior in the relationship between \(\Lambda\) and \(c_v\). At high stream powers, the sediment concentration seems to level off, whereas the volumetric concentration is reduced sharply as the stream power decreases. The Box-Lucas function

\[
c_v = c_{em}[1 - \exp(-k\Lambda)]
\]

has the following limiting characteristics:

\[
c_v \to c_{em} \quad \text{as} \quad \Lambda \to \infty
\]

and

\[
\frac{dc_v}{d\Lambda} = kc_{em} \quad \text{when} \quad \Lambda = 0
\]

The exponential function was found to describe the observed relationship between stream power and sediment concentration at the transport limit quite well. The parameter \(c_{em}\) represents the maximum possible sediment concentration. The product of \(k\) and \(c_{em}\) is the asymptotic value for \(F\) as the stream power approaches to zero. The fitted line (Fig. 2) has an adjusted \(R^2 = 0.95\), and estimated parameter values are \(c_{em} = 0.434 \pm 0.008\) and \(k = 0.115 \pm 0.006\). This implies that the \(F\) value approaches asymptotically to 0.05 as the stream power approaches to zero. Fig. 2 shows two constant values of \(F\) at 0.01 and 0.1 based on Eq. (14) with \(\Lambda_o = 0\), respectively, to place the observed relationship in perspective. From Eq. (21) and Fig. 2, it is clear that the fraction \(F\), i.e., the ratio of \(c_v\) over \(\Lambda\), would decrease as the stream power increases.

This trend of a decreasing fraction as stream power or sediment concentration increases is better presented in Fig. 3 which shows the fraction \(F\), computed using Eq. (12) with \(\Omega_o = 0\), as a function of the volumetric sediment concentration. It is evident that the fraction decreases essentially linearly as the concentration increases. Linear regression leads to

\[
F = 0.066 - 0.123c_v, \quad R^2 = 0.80
\]

The regression equation above implies a limiting fraction \(F\) of 0.066 when \(c_v = 0\), which is slightly higher than that estimated using the Box-Lucas Eq. (21). The regression equation also implies

\[
c_v = c_{em}[1 - \exp(-k\Lambda)]
\]
a limiting volumetric concentration of 0.537 when \( F = 0 \), which is yet again larger than that estimated using the Box-Lucas equation.

**Comparison with Other Formulas for Estimating Sediment Transport Capacity**

Fig. 4 shows the relationship between the stream power and the unit sediment discharge for this set of experiments. The line of best fit shown in Fig. 4 is given as follows:

\[
T_c = 0.470(\Omega - 0.905), \quad R^2 = 0.97 \tag{22}
\]

A very similar relationship was published in Zhang et al. (2009), and this relationship was the best of the five empirical relationships for the sediment transport capacity considered in Zhang et al. (2009). There are small differences in the coefficient, \( a \), and threshold stream power, \( \Omega_{th} \), between Eqs. (22) and (13) of Zhang et al. (2009). This occurred because of the way in which the unit sediment discharge was determined for this set of experiments (see the “Data and Method” section).

Using the generic relationship for sediment concentration at the transport limit, based on the energy required to sustain sediments in suspension or in motion, the following best fit power function is obtained:

\[
c_v = 0.0146 \left( \frac{V^3}{gD\omega} \right)^{0.5427}, \quad R^2 = 0.88 \tag{23}
\]

Fig. 5 shows a scatter plot of the volumetric concentration against \( V^3/(gD\omega) \). The mean settling velocity was based on \( d_{50} \), adjusted to take into account the effect of sediment concentration with Eq. (20). It is clear that the relationship is not linear in the log-log domain, and a power function of the form [Eq. (16)] is not appropriate for this set of experiments with steep slopes. It is also shown that the scatter plot suggests two linear segments with a break around \( V^3/(gD\omega) = 6,000 \) in the log-log domain. To use two linear segments in the log-log domain would result in a better fit; this, however, would incur a cost in terms of the number of parameters required. Instead of \( K \) and \( m \), there would be 5 parameters: one for the threshold and 2 for each of the linear segments.

Fig. 6(a) shows a comparison of the observed and modeled sediment concentration at the transport limit using Eq. (21) and fitted parameter values for \( c_{vm} \) and \( k \). The Nash-Sutcliffe coefficient of efficiency is 0.95 (Nash and Sutcliffe 1970). The agreement with the observations is excellent. Fig. 6(b) shows the same comparison using the empirical linear relationship between the sediment transport capacity expressed in terms of the unit sediment discharge and stream power as described by Eq. (22). The estimated transport capacity was then converted into volumetric concentrations using Eq. (2). The Nash-Sutcliffe coefficient of efficiency for Fig. 6(b) is 0.90. Finally, Eq. (23) was used to estimate the volumetric concentration, and a comparison with observed concentration is shown in Fig. 6(c). It is clear that Eq. (23) would overestimate the volumetric concentration for high sediment concentration (\( c_v > 0.4 \)). If Eq. (23) is used to estimate \( c_v \), the Nash-Sutcliffe coefficient of efficiency would be 0.79, much lower than the Box-Lucas function under two limiting conditions or the linear empirical relationship between stream power and unit sediment discharge. The ratio \( V^3/(gD\omega) \) ranged from \( 2.5 \times 10^5 \) to \( 6.3 \times 10^5 \), which is much higher than those commonly experienced in rivers (compare cf. Tables 4 and 5 in Shao and Wang 2005). This occurred largely because of the shallow water depth found in overland flows. Comparing Fig. 4(a) with Figs. 4(b and c), it is evident that the Box-Lucas function for \( c_v \) provides a tighter fit with the data and does not lead to negative estimated \( c_v \) values at very low stream power as the case with linear regression Eq. (22) above or the noticeable bias for high sediment concentrations with Eq. (23).

The formula widely used for the Yellow River of high sediment concentration was found to be not particularly useful. For 16 out of 64 slope-flow combinations, the formula is not applicable as the \( \ln[D/(6d_{50})] \) term in Eq. (17) would be negative because of the low flow depth. For the remaining 48 cases, the predicted \( c_v \) was grossly less than the observed values. The average observed \( c_v \) was 717 kg · m\(^{-3}\) for the remaining 48 measurements of sediment concentration at the transport limit, whereas the predicted \( c_v \) was only 1.82 kg · m\(^{-3}\) on average.

**Discussions**

Sediment transport capacity is commonly expressed in terms of the unit sediment discharge, e.g., Finkner et al. (1989), Zhang et al. (2009), and Ali et al. (2012). According to Eq. (2), it is clear that unit discharge is embedded in the sediment transport capacity when...
Fig. 6. Comparison of the observed and modeled sediment concentration at the transport limit: (a) with the Box-Lucas Eq. (21); (b) using linear regression between stream power and sediment transport capacity [Eq. (22)]; (c) the generic relationship between \( V^3/(gD_{\omega}) \) and volumetric concentration [Eq. (23)]

so expressed. At the same time, predictors of the sediment transport capacity such as sheer stress or stream power are closely related to the unit discharge as well. As such, the empirical relationships for the sediment transport capacity show generally good fit with observations because the unit discharge is involved on both sides of transport equations. Sediment concentration is a more fundamental quantity than the unit sediment discharge because concentration indicates the amount of sediments that can be carried per unit volume of water. It is only through a close examination of the sediment concentration at the transport limit with high flow rates and on steep slopes that its limiting behavior in relation to flow conditions becomes noticeable.

Eq. (21) offers a superior fit with the observed sediment concentration at the transport limit for this set of experiments. Comparison between the observed and modeled volumetric concentration shows an excellent agreement. It is worth noting that apart from the effect of sediment concentration on the density of water-sediment mixture in computing the stream power, i.e., Eqs. (6) and (18), and thus the dimensionless parameter, \( \Lambda \), that is related to the stream power, no other effects of sediment concentration have been explicitly considered. It is well documented that with sediments in suspension, especially when the concentration is high, there are appreciable effects because of the dampened turbulent intensity, reduced settling velocity, increased viscosity, and considerable vertical variations in sediment concentrations (Wan and Wang 1994; Winterwerp 2001; Hessel 2006). These effects, mostly observed in still water under lab conditions, may have been subdued at high stream power. Alternatively, these other effects of sediment concentration may have been embedded in the parameter \( k \) in the Box-Luca function [Eq. (21)].

The decreasing trend in the parameter \( F \) as the stream power increases is in contrast with experimental observations of Proffitt et al. (1993). Proffitt et al. (1993) conclude that a constant value of 0.1 is appropriate for the fraction of the stream power expended in maintaining sediments in suspension, and this constant value has been subsequently used for field applications and experimental study of multiclass sediment deposition with flow impoundment as a result of vegetation buffer strips (Rose et al. 2011; Hussein et al. 2007). By comparison with this set of experiments considered in this paper, the flow condition used by Proffitt et al. (1993) was much weaker and sediment concentration much smaller. The maximum stream power reached was about 1.8 W · m\(^{-2}\), which is about an order of magnitude less than the stream power considered in this paper. The observed sediment concentration (30–60 kg · m\(^{-3}\) or 0.015–0.03 by volume) was also considerably less than those measured in this set of experiments with concentration up to 1,245 kg · m\(^{-3}\). Because of the relatively low stream power, the full relationship between sediment concentration and the stream power-related parameter could not have been observed, although diagrams [Fig. 4 in Proffitt et al. (1993)] show a noticeable reduction in \( F \) from about 0.2 to 0.1 as the stream power increases. To some extent, their experimental results corroborate the decreasing trend in \( F \) with sediment concentration (Fig. 3) and the limiting behavior in the relationship between the volumetric concentration and the relative stream power developed in this paper.

Eq. (21) subject to two limiting constraints may provide a possible new framework to examine the effect of sediment concentration on transport capacity and prediction of transport capacity in general. When the parameter \( \Lambda \) is small and flow condition is relatively weak, the sediment transport is largely limited by the stream power available relative to characteristic particle size. When the parameter \( \Lambda \) is large and flow condition is relatively strong, the sediment concentration is only limited by the space available in flow to accommodate the sediments. This framework with a limiting behavior as shown in Fig. 7 provides additional opportunities to further examine the effect of sediment concentration on transport capacity for overland flows and possibly even for river flows in general.

In nature, there is a similar set of constraints placed on the mean evapotranspiration for a given amount of precipitation and the potential evapotranspiration. Budyko (1974) postulates that in dry areas where the potential evapotranspiration is much greater than precipitation, the actual evapotranspiration is limited by precipitation. Conversely, in the wet areas, actual
evapotranspiration is limited by the potential evapotranspiration. There is considerable renewed interest in the Budyko hypothesis as the first-order effect of climate on water balance and in extending the framework to include the effect of land use and vegetation for prediction of streamflows (Zhang et al. 2001, 2004). Similarity between Figs. 7 and 8 is clear in terms of the overall pattern of the relationship and the limiting behavior, although the functional form is different in details. Perhaps similar to the Budyko hypothesis in relation to the climate effect on evapotranspiration, the framework postulated in the paper probably has only highlighted the first-order effect of stream power and sediment concentration on the sediment transport capacity.

Conclusions

With the set of flume experiments involving steep slopes and high sediment concentrations, the paper developed a new functional relationship between stream power, settling velocity, and volumetric concentration at the transport limit and showed that stream power, once adjusted to take into account the effect of sediment concentration on the density of water-sediment mixture, could be used to estimate the volumetric concentration at the transport limit accurately. This new functional relationship with two asymptotic limits indicates that at low stream power, sediment concentration is limited by the stream power available to sustain sediments in suspension or in motion, whereas at the high stream power, sediment concentration is limited by the maximum possible sediment concentration, likely to be dictated by the space available to accommodate sediments in the flow. This new relationship is superior to empirical relationships based on stream power and other commonly used predictors of the transport capacity of rivers. The paper has also shown that formulas for transport capacity well tested for rivers with high sediment concentrations could not be used to predict the transport capacity of overland flow accurately.

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Notation

The following symbols are used in this paper:

- $a$ = coefficient in an empirical equation for $T_e(s^2 \cdot m^{-1})$;
- $c_i$ = sediment concentration for size class $i$(kg · m$^{-3}$);
- $c_r$ = sediment concentration at the transport limit, an alternative measure of the transport capacity (kg · m$^{-3}$);
- $c_{vm}$ = volumetric concentration, also called relative concentration at the transport limit;
- $c_{vm}$ = maximum volumetric concentration;
- $D$ = depth of flow (m);
- $d$ = particle size (m);
- $d_{sp}$ = median particle size (m);
- $d^i$ = dimensionless particle parameter;
- $d_i$ = rate of deposition per unit area for size class $i$(kg · m$^{-2}$ · s$^{-1}$);
- $F$ = parameter to indicate the fraction of energy expended on erosion and sediment transport;
- $G$ = sediment discharge (kg · s$^{-1}$);
- $g$ = acceleration due to gravity (9.81) (m · s$^{-2}$);
- $H$ = parameter relating to the fraction of deposited sediments covering the eroding surface;
- $J$ = specific energy of entrainment (J · kg$^{-1}$);
- $K$ = coefficient in the generic transport equation;
- $M_{dp}$ = mass of deposited sediments in size class $i$(kg · m$^{-2}$);
- $M_{ds}$ = total mass of deposited sediments (kg · m$^{-2}$);
- $M_r$ = mass of sediments collected in a sample (kg);
- $m$ = exponent in the generic transport equation;
- $N$ = number of size classes;
- $Q$ = water discharge (m$^3$ · s$^{-1}$);
- $q$ = water discharge per unit width of flow, simply called unit discharge (m$^2$ · s$^{-1}$);
\[ q_s = \text{sediment discharge per unit width of flow, simply called unit sediment discharge (kg \cdot m^{-1} \cdot s^{-1});} \]
\[ R^2 = \text{correlation coefficient squared, also called coefficient of determination;} \]
\[ r_i = \text{rate of entrainment per unit area (kg \cdot m^{-2} \cdot s^{-1});} \]
\[ r_{re} = \text{rate of reentrainment per unit area (kg \cdot m^{-2} \cdot s^{-1});} \]
\[ S = \text{tangent of slope angle;} \]
\[ T_c = \text{sediment transport capacity expressed in terms of unit sediment discharge, i.e., maximum } q_s \text{ for given flow condition and sediment characteristics (kg \cdot m^{-1} \cdot s^{-1});} \]
\[ t = \text{time (s);} \]
\[ V_s = \text{volume of sediments in a sample (m^3);} \]
\[ V_w = \text{volume of water in a sample (m^3);} \]
\[ x = \text{distance downslope (m);} \]
\[ \alpha_i = \text{parameter relating to the nonuniform distribution of sediment concentration in the vertical direction for size class } i; \]
\[ \kappa = \text{von Kármán’s constant;} \]
\[ \kappa_0 = \text{von Kármán’s constant for clear water;} \]
\[ \Lambda = \text{dimensionless parameter in relation to stream power, submerged weight, water depth, and average settling velocity; also called relative stream power;} \]
\[ \Lambda_a = \text{threshold relative stream power, below which the transport capacity would be zero;} \]
\[ \nu = \text{kinematic viscosity (10^{-6})(m^2 \cdot s^{-1});} \]
\[ \rho = \text{water density (1000) (kg \cdot m^{-3});} \]
\[ \rho_m = \text{density of water-sediment mixture, also called effective density (kg \cdot m^{-3});} \]
\[ \rho_s = \text{sediment density (kg \cdot m^{-3});} \]
\[ \Omega = \text{stream power per unit area, often simply called stream power (W \cdot m^{-2});} \]
\[ \Omega_t = \text{threshold stream power per unit area (W \cdot m^{-2});} \]
\[ \omega = \text{settling velocity (m \cdot s^{-1}); and} \]
\[ \omega_a = \text{average settling velocity (m \cdot s^{-1}).} \]

**References**


