Modified CN Method for Small Watershed Infiltration Simulation

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Abstract: Infiltration is an essential process in watershed hydrology. The curve number (CN) method has been widely used to calculate watershed infiltration for a given rainfall input but does not consider steady infiltration. This study develops a modified CN (MCN) method that included a term for the steady infiltration amount ($F_c$). Observed rainfall–runoff data for 14 rainfall events from a typical small watershed on the Loess Plateau of China were used to derive the watershed final infiltration rate ($f_c$). Watershed infiltration after runoff initiation was then calculated by both the MCN and CN methods using initial abstraction values that were either observed ($I_{a-obs}$) or calculated ($I_{a-0.25}$) based on the calibrated $f_c$. Three criteria [relative error ($E_r$), model efficiency coefficient ($E$), and root mean square error (RMSE)] and visual assessments of graphed results were used to evaluate the methods’ simulation performances. The MCN method generally outperformed the CN method when using either of the initial abstractions for infiltration calculations. Moreover, infiltration calculated by the MCN method was more accurate when using $I_{a-obs}$ ($E = 0.9$; RMSE = 24.4%); than when using $I_{a-0.25}$ ($E = 0.8$; RMSE = 30.8%). The CN method using either initial abstraction tended to underestimate infiltration, especially higher values, which were always lower than the MCN method estimates. In addition, watershed runoff was calculated by both methods using only $I_{a-obs}$. The CN method ($E = 0.4$ and RMSE = 78.1%; excluding 3 outliers) outperformed the CN method ($E = -23.8$ and RMSE = 506.8%; excluding 3 outliers) but not to the same extent as when calculating infiltration. The theoretical analyses and practical application results indicated that the MCN method is more appropriate than the CN method for predicting infiltration in small watersheds on the Loess Plateau. DOI: 10.1061/(ASCE)HE.1943-5584.0001125. © 2014 American Society of Civil Engineers.

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Introduction

Infiltration is of great importance due to its fundamental role in rainfall–runoff processes (Liu and Wei 1989; Yu and Chen 1989), the hydrologic cycle by linking land-surface runoff and groundwater recharge (Mishra et al. 2013), irrigation, and agriculture (Mishra et al. 2003). During the course of a natural rainstorm, initial soil infiltrability is usually higher than the rainfall intensity and all the rainwater enters the soils. As the rainstorm proceeds, soil infiltrability decreases until the infiltration rate becomes lower than the rainfall intensity, at which point runoff commences. When the rainstorm is of sufficient duration, infiltration finally attains a relatively stable value, i.e., the final infiltration rate, which is the minimum rate at which water enters into the soil (Mbagwu 1993). The processes described above are typical Hortonian infiltration processes (Hillel 1998). The final infiltration rate is dependent on various factors including the rainfall intensity (Li and Shao 2004), the soil surface, and antecedent moisture conditions (Liu et al. 2009).

In the 1950s, the Curve Number method was proposed by the United States Department of Agriculture (USDA) as a means of predicting the runoff response to rainfall events [Soil Conservation Service (SCS) 1972]. Due to its simplicity and stability, the CN method has been used worldwide as a conceptual tool to predict runoff response, taking into account several watershed characteristics such as soils, land use and management, and antecedent soil moisture (Mishra et al. 2005; Ponce and Hawkins 1996). The CN method has also been applied in a number of commonly used hydrologic and sediment yield models such as erosion-productivity impact calculator (EPIC) (Sharpley and Williams 1990), soil and water assessment tool (SWAT) (Arnold et al. 1996), agricultural non-point source (AGNPS) (Young et al. 1989), and chemical runoff and erosion from agricultural management systems (CREAMS) (Knisel 1980) as well as in many others (e.g., Du et al. 2009; Kliment et al. 2008; Melesse and Graham 2004; Tyagi et al. 2008; Bhunya et al. 2010). The CN method has also been used to estimate soil water content (Reshmidiev et al. 2008).

The event-based CN method comprises one water balance equation and two fundamental hypotheses (SCS 1972). The first
hypothesis equates the ratio of the amount of direct surface runoff \( (Q) \) to the maximum potential surface runoff \( (P - I_a) \) with the ratio of the amount of infiltration \( (F) \) to the amount of the potential maximum retention \( (S) \). The second hypothesis relates the initial abstraction \( (I_p) \) to the maximum retention. Thus, the CN method consists of the following equations:

\[
P = I_a + F + Q \tag{1a}
\]

\[
\frac{Q}{P - I_a} = F \tag{1b}
\]

\[
I_a = \lambda S \tag{1c}
\]

where \( P \) = rainfall, mm; \( I_a \) = initial abstraction, mm; \( F \) = cumulative infiltration after runoff initiation, mm; \( Q \) = runoff, mm; \( S \) = watershed potential maximum retention after runoff initiation, mm; and \( \lambda \) = initial abstraction ratio, dimensionless. Typical values of \( \lambda \) range from 0 to 0.3 (Ponce and Hawkins 1996), and the standard value is 0.2 (SCS 1972). The initial abstraction consists of evaporation, canopy interception, and surface storage as well as the cumulative infiltration before runoff initiation (Ponce and Hawkins 1996). Combining Eqs. (1a) and (1b), the most conventional forms of estimations for runoff and infiltration after runoff initiation by the CN method are given as

\[
Q = \frac{(P - I_a)^2}{P - I_a + S} \quad P > I_a \tag{2a}
\]

\[
F = \frac{(P - I_a)S}{P - I_a + S} \quad P > I_a \tag{2b}
\]

The parameter \( S \) is defined by

\[
S = \frac{25400}{CN} - 254 \tag{3}
\]

where \( S \) is in mm, and \( CN \) = curve number, determined by watershed characteristics including soils, land use, hydrologic conditions, etc. The CN values are differentiated into CNI, CNII, and CNIII for antecedent soil moisture conditions AMCI (dry), AMCII (normal), and AMCIII (wet), respectively, which are based on the amount of precipitation occurring in the preceding 5 days.

The CN method has been the basis for the development of other models and the source of some controversy. Aron et al. (1977) developed an infiltration equation using a differential form of the CN method; however, the steady infiltration phase of the rainfall event was ignored. Hjelmfelt (1980) suggested that the potential maximum retention did not include the initial abstraction and then developed a differentiated infiltration equation based on the CN method. Hjelmfelt’s infiltration equation was identical to the Holtan–Overton equation (Holtan 1961; Overton 1964) for the particular case of a storm with constant rainfall intensity and a zero final infiltration rate, which in actuality ignored the steady infiltration phase. Chen (1982) claimed that the parameter \( S \) included the initial abstraction, \( I_a \), and that the Holtan–Overton equation should be combined with the CN method. He suggested a term for the final infiltration rate to improve Hjelmfelt’s method, while deriving the linear relationship between \( I_a \) and \( S \) through regression analysis, maintaining that the parameter \( S \) did not include the initial abstraction (SCS 1972). The Mockus infiltration method derived in 1949 by Mockus (1949) was based on the Horton equation (Horton 1933) and the CN concept of proportionality (Mishra and Singh 2004). Mishra and Singh (2004) presented a detailed derivation of the Mockus method in their study and pointed out that the total infiltration was underestimated by the Mockus method due to the absence of a term for the steady infiltration phase. Mishra and Singh (2004) then developed an improved infiltration method that was based on their theoretical analysis of the CN method. It combined the CN method and the Horton infiltration equation (Horton 1933), took the steady infiltration phase into account, and accurately fitted the measured data. The Mishra and Singh equation is given by

\[
f = f_c + \frac{i - f_c}{(1 - \lambda + k) t^2} \tag{4}
\]

where \( f \) = infiltration rate (mm h\(^{-1}\)) at time \( t \), and time \( t \) is measured from the beginning of the infiltration; \( f_c \) = final infiltration rate (mm h\(^{-1}\)); \( i \) = rainfall intensity (mm h\(^{-1}\)); \( k \) = Horton decay constant (h\(^{-1}\)); and \( \lambda \) = initial abstraction ratio, dimensionless, which has the same function as in the CN method. However, this infiltration method was based on simulated rainstorm events that had constant rainfall intensities, which is not the case in most natural rainfall events. Furthermore, the Mishra and Singh equation needs two more parameters than the CN method, i.e., the final infiltration rate \( (f_c) \) and the Horton decay constant \( (k) \).

Although the CN method has been widely used and/or modified for rain–runoff simulation in the watersheds on the Loess Plateau of China (e.g., Luo et al. 2002; Liu et al. 2005; Huang et al. 2006, 2007), little attention had been paid to the simulation of the watershed infiltration during natural rainfall events.

Therefore, the objectives of this study were: (1) to modify the CN method to simulate watershed infiltration after runoff initiation by introducing a term for the final infiltration rate based on theoretical analyses; (2) to test the simulation performance of the newly developed infiltration method by the observed and calculated initial abstraction data for the study watershed; and (3) to further discuss the implications and applications of the modified method for watershed runoff simulation.

**Model Developments**

**CN Infiltration Method**

Differentiation of Eq. (2b) with respect to \( t \) leads to

\[
f(t) = \frac{S^2}{(P - I_a + S)^2} i(t) dt \quad P > I_a \tag{5}
\]

where \( f(t) \) = infiltration rate, mm h\(^{-1}\); \( i(t) \) = rainfall intensity, mm h\(^{-1}\); \( t \) = rainfall duration, h; and \( P \) = \( i(t) t \), mm, under a constant rainfall intensity.

According to Eq. (5), \( f(t) \) is the function of \( S, P, I_a \) and \( i(t) \). As time \( t \) approaches infinity and \( P \) approaches infinity, the infiltration rate approaches zero. Therefore, the absence of the steady infiltration term leads to an underestimation of infiltration by this infiltration method (Mishra and Singh 2004; Michel et al. 2005).

**Development of the Modified CN (MCN) Method**

Since the CN method does not incorporate any estimate of steady infiltration, the main purpose of the MCN method was to correct this by introducing the amount of steady infiltration \( (F_c) \), mm. The water balance equation and the second hypothesis of the MCN method are the same as those of the CN method. However, a change was introduced for the first hypothesis in that the ratio of the actual runoff \( (Q) \) to the potential runoff \( (P - I_a) \) is equal to the ratio of the changes in infiltration \( (F - F_c) \) after runoff initiation to the potential maximum retention \( (S) \). Three basic assumptions have been made for the MCN method: (1) the method is only adapted to the
condition of Hortonian infiltration and runoff, as is the CN method; (2) the watershed final infiltration rate is considered to be uniform across an entire watershed during a rainstorm; and (3) the rainfall intensity is higher than the final infiltration rate \( i(t) \) throughout the rainfall event, which ensures a continuous infiltration process with an infiltration rate higher than or equal to the final infiltration rate.

Thus, the modified CN (MCN) method is expressed as

\[
P = I_a + F + Q \quad (6a)
\]

\[
\frac{Q}{P - I_a} = \frac{F - F_c}{S} \quad (6b)
\]

\[
I_a = \lambda S \quad (6c)
\]

Fig. 1 presents a schematic of the MCN method. In Fig.1, runoff \( Q \) appears after the initial abstraction \( I_a \) during the rainfall course. After runoff initiation, the rainfall amount falling on the ground is separated into runoff \( Q \), the time-dependent infiltration \( F_d \), and steady infiltration \( F_c \). Combining Eq. (6a) with Eq. (6b) leads to

\[
Q = \frac{(P - I_a)(P - I_a - F_c)}{P - I_a + S} \quad P > (I_a + F_c) \quad (7a)
\]

\[
F = \frac{(P - I_a)(F_c + S)}{P - I_a + S} \quad P > (I_a + F_c) \quad (7b)
\]

In the MCN method, runoff occurs only when cumulative rainfall is higher than the sum of the initial abstraction and the steady infiltration, which is an essential divergence from the CN method. When compared with Eq. (2a), Eq. (7a) yields a lower value of \( Q \). The differences in \( Q \) and \( F \) between the CN and MCN methods, i.e., \( \Delta Q \) and \( \Delta F \), are derived from Eqs. (7a) and (2a), and Eqs. (7b) and (2b), respectively, to give

\[
\Delta Q = \frac{(P - I_a)(P - I_a)}{P - I_a + S} - \frac{(P - I_a)(P - I_a - F_c)}{P - I_a + S} \quad P > (I_a + F_c) \quad (8a)
\]

\[
\Delta F = \frac{(P - I_a)(F_c + S)}{P - I_a + S} - \frac{(P - I_a)S}{P - I_a + S} - \frac{(P - I_a)F_c}{P - I_a + S} \quad P > (I_a + F_c) \quad (8b)
\]

Clearly, from Eqs. (8a) and (8b), \( \Delta Q \) equals \( \Delta F \), indicating that the lower estimates of runoff by the MCN method are equal to the lower infiltration estimates by the CN method, which naturally results from maintaining the water balance. When \( F_c \) is equal to zero, both \( \Delta Q \) and \( \Delta F \) are also equal to zero, indicating that the CN method is the special case of the MCN method for zero steady infiltration. However, in the MCN method when \( t \) approaches infinity, \( P \) approaches infinity and, therefore, both \( \Delta Q \) and \( \Delta F \) approach the steady infiltration, \( F_c \), which is the fundamental difference between the MCN and CN methods.

Differentiation of Eq. (7b) with respect to time, \( t \), gives

\[
f(t) = \frac{f_c(P - I_a) + (f_cT + S)S}{P - I_a + S} \left( \frac{P - I_a + S}{P - I_a + S} \right) dt \quad P > (I_a + F_c) \quad (9)
\]

In Eq. (9), \( t \) = rainfall duration, \( h \); \( f_c = \) infiltration duration, which is the time from runoff initiation to the end of the rainfall event; \( h \); and \( f_c = \) final infiltration rate, mm h\(^{-1}\). At the beginning of the rainfall event, when \( t, P, \) and \( I_a \) all equal zero, \( f(t) \) equals \( i(t) \). When \( t \) approaches the moment of runoff initiation, \( P \) approaches \( I_a \), and \( f(t) \) is still equal to \( i(t) \). As \( t \) approaches infinity, the first term on the right-hand side of Eq. (9) approaches \( f_c \) while the second term approaches zero and, therefore, \( f(t) \) approaches the extreme value of \( f_c \). The theoretical analyses of these three characteristic temporal points in a rainfall event indicate that the MCN method is consistent with reality. Notably, the MCN method is also rational when applied to rainfall events with fluctuating rainfall intensities.

**Data Collection**

**Study Watershed**

Observed rainfall–runoff data from a typical small watershed, Qiaozui–West watershed (Fig. 2), nested in the Luoyugou (LYG) watershed in the hilly–gully region on the Loess Plateau, were used first to calibrate the watershed final infiltration rate and then to test the simulation performance of the MCN method. In the study area, storm rainfall events are generally of short duration and high intensity. Furthermore, the soils are generally very deep, which prevents groundwater participating in the processes of soil water vertical circulation and runoff initiation. Therefore, Hortonian infiltration excess runoff theory is entirely appropriate for the study area (Tang et al. 1997; Huang et al. 1999).

The total watershed infiltration after runoff initiation was determined using Eq. (1a). Rainfall data were obtained from three rain gauges in the upper part, close to the middle, and downstream of the watershed. The recorded intervals of the rainfall process varied from 2 minutes to several hours during rainfall events. Total runoff from the watershed was measured at a hydrometric station situated at the main water channel outlet in the south.

The catchment area of the study watershed is 1.14 km\(^2\). The main water channel is 2.2 km long and has an elevation range of 1,330–1,707 m. There were four main land use types in the watershed consisting of farmland (mainly on slopes), grassland, residential areas, and bare soils (Table 1). The main soil type is a black cinnamonic soil, which covers most of the watershed. Since the watershed has been used as the control in a study of soil and water conservation methods, it was assumed that land use changes did not occur in the study watershed during the study years. Accordingly, the watershed \( CN \) values for the dry, normal, and wet antecedent moisture conditions were calculated as 65, 82, and 88, respectively (Zhou and Lei 2011).
**Derivation of the Watershed Final Infiltration Rate**

In this study, two alternative approaches were adopted to determine watershed initial abstraction. The first approach was to use observed initial abstraction. Since the study watershed was small in area (≈ 1 km$^2$), the travel time for runoff to reach the outlet could be neglected. The cumulative rainfall amount at the time when the first increase in discharge at the outlet was observed was taken to be the observed initial abstraction, which was denoted as $I_{a-obs}$. The second approach used the regression equation, $I_a = 0.2S$ (SCS 1972), to calculate the initial abstraction, which was denoted as $I_{a-0.2S}$. The watershed steady infiltration ($F_c$) for each studied event was back-calculated using Eq. (10), which was derived from Eq. (7a).

$$F_c = P - I_a - \frac{Q(P - I_a + S)}{P - I_a} \quad P > (I_a + F_c) \quad (10)$$

The steady infiltration duration ($t_f$) was derived from the rainfall duration ($t$) minus the initial abstraction duration ($t_{ip}$). The final infiltration rate ($f_c$) was then derived using Eq. (11)

$$f_c = \frac{F_c}{t_f} \quad P > (I_a + F_c) \quad (11)$$

**Evaluation of the Simulation Performance of the MCN Method**

Using the derived watershed final infiltration rate, the total infiltration ($F$) after runoff initiation for each rainfall event was calculated using Eq. (7b). Three mathematical criteria were used to evaluate the derived watershed steady infiltration simulation, as well as the simulation performance of the MCN method. First, the relative error ($E_r$) was defined as

$$E_r = \frac{S_i - O_i}{O_i} \times 100\% \quad (12)$$

where $S_i$ and $O_i$ = simulated and observed values, respectively.

Second, the model efficiency coefficient ($E$) was defined by Nash and Sutcliffe (1970) as

$$E = 1 - \frac{\sum_{i=1}^{n}(O_i - S_i)^2}{\sum_{i=1}^{n}(O_i - \bar{O})^2} \quad (13)$$

where $\bar{O}$ = mean of the observed values; and $n$ = number of observed events.

An $E$ value of one indicates perfect agreement between observed and simulated values, while declining values, until zero, indicate increasingly poorer agreement. A negative value for $E$ indicates that the mean of the observed value gives a better estimate than the simulated values (Esteves et al. 2000). The third criterion was the root mean square error (RMSE) (Loague and Green 1991), which is increasingly being used for comparisons and evaluations of simulation models (Addiscott et al. 1995) and is expressed as

$$\text{RMSE} = \frac{100}{\bar{O}} \sqrt{\frac{\sum_{i=1}^{n}(S_i - O_i)^2}{n}} \quad (14)$$

**Results and Discussion**

**Watershed Infiltration Simulation after Runoff Initiation by the MCN and CN Methods**

Watershed Infiltration after Runoff Initiation Determined by the Observed Initial Abstraction

The observed rainfall–runoff data from 14 rainfall events (Table 2), which have complete information on characteristic temporal points, including those of runoff initiation during the rainfall events, were used to derive the watershed final infiltration rate. Using the median value improved the model simulation performance over that when
using the mean, maximum, or minimum values of the final infiltration rate, the median value calculated using observed initial abstractions (Ia_obs) of the watershed was 4.8 mm h⁻¹.

The maximum and minimum relative errors (Er) of the infiltration simulated by the MCN method using the observed initial abstraction (Ia_obs) were 46.8% (Event No. 5) and 1.1% (Event No. 4). Event No. 5 occurred under the AMCI condition (dry) and was of long duration (33.7 h) with a low mean rainfall intensity (1.9 mm h⁻¹), resulting in a total rainfall amount of 65.5 mm. In contrast, Event No. 4 occurred under the AMCIII condition (normal), and the duration was only 1.3 h, delivering a total rainfall amount of 18.4 mm; thus the mean rainfall intensity was 14.2 mm h⁻¹, which was much higher than that of Event No. 5.

The possible key factor that affected simulation performance in these cases was the rainfall intensity. The higher the mean rainfall intensity, the more likely the assumption of the MCN method, that the rainfall intensity should be higher than the final infiltration rate, would be met throughout the rainfall event.

In contrast, for the CN simulations using the observed initial abstraction (Ia_obs), the antecedent moisture content may have been the key factor. The maximum and minimum relative errors (Er) of the infiltration simulations were −55.2% (Event No. 8) and −0.3% (Event No. 6) (Table 3). Event No. 8 occurred under the AMCIII condition (wet), and had a duration of 7.9 h, delivering a total rainfall amount of 54.2 mm with a mean rainfall intensity of 6.9 mm h⁻¹. In contrast, Event No. 6 was under the AMCI condition (dry) and had a smaller duration of 4.2 h and a total rainfall amount of 23.7 mm, with a mean rainfall intensity of 5.6 mm h⁻¹. In these two events, the mean rainfall intensities were similar, suggesting that the possible key factor may have been the antecedent moisture conditions, which were notably different, i.e., dry and wet. Hortonian infiltration excess runoff, which is the basis of the MCN method, should be the main mechanism producing runoff on the Loess Plateau based on the characteristics of the Plateau. The possibility of Hortonian excess infiltration runoff occurring is greater under dry than under wet antecedent conditions. Therefore, this would account for the higher simulation accuracy for the event under the AMCI condition than that under the AMCIII condition.

Moreover, all the relative errors for the CN method were negative, which suggested that the CN method consistently underestimated the infiltration for all of the studied events. The model efficiency coefficient (E) value of the MCN method was higher (0.9) than that of the CN method (0.6), which indicates that the MCN method was more efficient than the CN method. The RMSE value of the MCN method was 24.4%, while that of the CN method was 53.5%, also indicating that the MCN method was more accurate than the CN method.

Both the MCN and CN methods were able to accurately simulate the infiltration for small infiltration events (Fobs < 14 mm) (Fig. 3); while for relatively large infiltration events (Fobs > 14 mm), the MCN method was generally the model with the better performance. This was probably due to the increasing tendency of the CN model to underestimate infiltration as the storm length increased because it estimated infiltration rate changes based on approaching a final infiltration rate value of zero, whereas the MCN model was less likely to do so since it approached a realistic final infiltration value. The underestimation of infiltration values simulated by the CN method using the observed initial abstraction (Ia_obs), as also indicated in Table 3, can be clearly seen in Fig. 3 where all of the simulated values were scattered under the line of perfect fit.
Watershed Infiltration after Runoff Initiation Determined by the Calculated Initial Abstraction

The initial abstraction was calculated using the regression equation between \( I_a \) and \( S \) (\( I_a = 0.2S \)), and then the calculated \( I_a \) (\( I_{a-0.25} \)) was used to back-calculate the watershed final infiltration rates using Eqs. (10) and (11). There were 5 out of the 14 studied rainfall events (Event Nos. 1, 6, 11, 12, 13) that could not satisfy the preconditions of either the MCN or CN methods due to the aberrations in the calculated \( I_a \) values, which could even predict negative runoff. This was mainly because the calculated \( I_a \) values did not represent the real initial abstraction in the watershed as accurately as the observed \( I_a \) values did. Firstly, it was likely that the error introduced for the observed value was less than that for the calculated one since the latter includes errors from: (1) making an accurate assessment of the value of \( S \), which involves a complex estimation based on the spatial variability of several factors within the watershed; and (2) assuming that the statistical relationship \( I_a = 0.2S \) derived for a large watershed dataset in United States, is both valid and applicable to the study area (and elsewhere), which has been frequently questioned (e.g., Ponce and Hawkins 1996; Shi et al. 2009; Zhou et al. 2011). Therefore, in this study, neither the watershed infiltration nor the watershed final infiltration rate could be calculated for these five events.

The median value of the back-calculated watershed final infiltration rates determined from the remaining nine studied rainfall events was 4.2 mm h\(^{-1}\), which was less than that derived from all 14 events when using the observed initial abstraction data (4.8 mm h\(^{-1}\)). The maximum and the minimum relative errors (\( E_r \)) of the infiltration simulated by the MCN method using the calculated initial abstraction (\( I_{a-0.25} \)) were 40 and \(-0.03\%\); while those for the CN method were \(-55.6\%\) and \(-7.6\%\), respectively (Table 3). Similar to the simulation results using the observed initial abstraction (\( I_{a-obs} \)), the simulation results from the CN method using the calculated initial abstraction (\( I_{a-0.25} \)) tended to underestimate all of the infiltration values (i.e., all the \( E_r \) values were negative). The model efficiency coefficient (\( E \)) value of the MCN method was 0.8, which was higher than the value of 0.5 for the CN method; however, these \( E \) values were both lower than their respective values for the MCN and CN methods (0.9 and 0.6) obtained when using the observed initial abstraction (\( I_{a-obs} \)). This indicated that the use of the calculated initial abstraction (\( I_{a-0.25} \)) reduced model efficiency. The RMSE value of the simulation results obtained from the MCN method using the calculated initial abstraction (\( I_{a-0.25} \)) was 30.8\%, while that obtained when using the CN method was 56.0\%; and both of these values were higher than the respective RMSE values of the simulation results obtained by the two methods when using the observed initial abstraction (\( I_{a-obs} \)). This is further proof that the use of the calculated initial abstraction (\( I_{a-0.25} \)) led to inaccuracy in the simulation.

All of the infiltration values simulated by the CN method using the calculated initial abstraction (\( I_{a-0.25} \)) were scattered under the line of perfect fit (Fig. 4), which was consistent with the results shown in Table 3. This is similar to the pattern shown in Fig. 3, where both methods used the observed initial abstraction (\( I_{a-obs} \)). The MCN method generally outperformed the CN method when using the calculated initial abstraction (\( I_{a-0.25} \)) (Table 3, Fig. 4). All of the infiltration values simulated by the CN method were lower than those simulated by the MCN method. These findings were similar to those obtained when the methods used the observed initial abstraction (\( I_{a-obs} \)). Moreover, the simulated results demonstrate that the MCN method performed better than the CN method when using either the observed (\( I_{a-obs} \)) or the calculated (\( I_{a-0.25} \)) initial abstractions, especially for the relatively higher infiltration values; and the MCN method simulation results were more accurate when using the observed initial abstraction (\( I_{a-obs} \)) than when using the calculated initial abstraction (\( I_{a-0.25} \)).

Soil infiltration ability can be affected by the canopy over the soil surface, the antecedent soil moisture condition, soil structure, soil texture, etc. Within a watershed, spatial variations in the soil conditions are common and these may differ for different rainfall events, i.e., they may demonstrate temporal variation. Thus, using a unique value such as the watershed infiltration rate imposes bias effects on the simulation results. Therefore, the simulation results of the MCN method were sensitive to the determined watershed infiltration rate. The determined watershed infiltration rate in this study was lower (the lowest value is 18 mm h\(^{-1}\)) than those measured under simulated rainfall conditions in a small watershed on the Loess Plateau (Yuan and Jiang 2001). The reasons for this may be due to differences between the determined and measured scales in the two studies: Yuan and Jiang (2001) measured the infiltration...
rates at different sites within the watershed that were under various land uses, whereas in this study the infiltration rate was determined considering the whole watershed. In addition, the soil textures in the two cited studies differed in that Yuan and Jiang (2001) had coarser textured soils. Furthermore, there are differences between simulated and natural rainfall events. Simulated rainfall events are usually characterized by uniform rainfall intensity while rainfall intensity changes during natural events. Therefore, rainfall intensities might be lower than the soil infiltrability at certain times during natural storms. In such cases, the derived watershed infiltration rate is dependent on rainfall intensity, and this would be different from the infiltration rates measured under rainfall intensities that always exceed soil infiltrability once runoff has commenced (Jiang et al. 1990; Wang et al. 1991). When the infiltration process is interrupted due to rainfall intensities lower than the final infiltration rate, including temporary complete cessation of the rainfall during a relatively long rainfall event, the violation of the assumption of a continuous infiltration process whereby the infiltration rate is always higher than or equal to \( f_c \) would lead to a lower calculated watershed final infiltration rate.

### Watershed Runoff Simulation by the MCN and CN Methods

Watershed runoff for the 14 studied rainfall–runoff events was calculated by both the MCN and CN methods using the observed initial abstraction \( (I_{a,obs}) \) data to avoid the inaccuracy of the statistical relationship between \( I_a \) and \( S \). The determined final infiltration rate of 4.8 mm h\(^{-1}\) was used in the MCN method. Three runoff events were considered to be outliers (two that were negative, one that was negligible) as derived by the MCN method. This occurred because the final infiltration rate used (4.8 mm h\(^{-1}\)) was higher than the back-calculated final infiltration rates for these three events \( (f_c \) was 0.8, 1.3 and 0.1 mm h\(^{-1}\) for Event Nos. 1, 5, and 6, respectively). Runoff simulation results of the studied rainfall events (all 14 events and the 11 events having discarded the three outliers) are evaluated in Table 4 and are presented in Fig. 5.

The simulation results produced by the MCN method for the 11 studied events were better than those of all 14 studied events, which included the three outliers, in terms of the increased model efficiency coefficient \( (E) \) and the reduced root mean square error (RMSE). In contrast, the simulation results obtained using the CN method for the 14 studied events were better than those of the 11 studied events, which excluded the three outliers. Furthermore, the MCN method simulations outperformed those of the CN method for all the studied events (14 events), and especially so when the three outliers were discarded (11 events). In Fig. 5, the observed runoff \( (O_i) \) versus simulated runoff \( (S_i) \) by the two methods is fitted by linear regression equations for the 11 rainfall–runoff events. The predictions of the MCN method were more accurate as indicated by the slope value that was closer to 1 and the intercept value that was closer to zero than the corresponding values for the CN method.

### Table 4. Evaluation of Runoff Simulation Results Obtained by the Modified Curve Number (MCN) and Curve Number (CN) Methods Using the Observed Initial Abstractions \( (I_{a,obs}) \)

<table>
<thead>
<tr>
<th>Models</th>
<th>14 events</th>
<th>11 events</th>
<th>14 events</th>
<th>11 events</th>
</tr>
</thead>
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<tr>
<td>MCN</td>
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<td>0.4</td>
<td>156.8</td>
<td>78.1</td>
</tr>
<tr>
<td>CN</td>
<td>4.2</td>
<td>-23.8</td>
<td>344.5</td>
<td>506.8</td>
</tr>
</tbody>
</table>

Note: \( E \), Nash-Sutcliffe model efficiency coefficient; \( RMSE \), root mean square error; Events refer to the number of event included in the analysis since three outliers were either included or excluded.

Furthermore, the relationship between predicted values and observed values was more consistent for the MCN method, as indicated by the determined coefficient value (0.88), which was higher than that for the CN method (0.58). This supports the evaluation of the models reported in Table 4.

### Possible Factors Affecting Watershed Infiltration Simulation

As the MCN method introduces steady infiltration, runoff would decrease due to the increase in the infiltration under a given rainfall in accordance with Eq. (1a). The increase in infiltration equals the decrease in runoff based on Eqs. (8a) and (8b). Fig. 6 shows the symmetrical changes in the differences in infiltration and runoff.

![Fig. 5. Runoff simulation performance of the modified curve number (MCN) and curve number (CN) methods using the observed initial abstraction \( (I_{a,obs}) \) for 11 studied rainfall events \( (S_i \) and \( O_i \) are the simulated and observed runoff values, respectively)](image)

![Fig. 6. Changes in the simulated infiltration after runoff initiation and runoff when comparing the modified curve number (MCN) and curve number (CN) methods for 11 studied rainfall events](image)
when derived by the two different methods, MCN and CN, for the 11 studied rainfall–runoff events.

As mentioned above, many factors can affect the watershed final infiltration rate, e.g., the rainfall intensity, which should not only consider the mean rainfall intensity but also the variation of intensities within a rainstorm event, is a factor affecting runoff production, especially when comparing an advanced storm (highest intensities at the start) with a delayed storm (highest intensities at the end). Another point to consider is that the initiation of runoff can be affected by the land use conditions, although the study watershed covered only a small area (about 1 km²). Thus, using a weighted watershed CN could be misleading when predicting when or if runoff would commence, i.e., one part of the watershed under one land use may produce runoff while another part under a different land use would not. Moreover, the error in simulation results might be exacerbated by the assumption that the final infiltration rate is uniform across an entire watershed during a rainstorm, which is unlikely to be the case given the spatial variability of the affecting factors. The value of $f_c$ is deemed to be one of the watershed properties and is mainly affected by the soil textures in the study region. In this study area, it would not be significantly affected by the AMC condition due to the low clay content and high silt content (Lado et al. 2004). However, where soils have high clay contents (e.g., >60%), the reductions in slaking and increases in aggregate stability when the AMC is wet rather than dry would likely have great effects on the whole infiltration process including the $f_c$. Therefore, in areas where clay contents are high, this effect should be taken into consideration.

During a natural rainfall event, the infiltration rate is related to but never higher than the rainfall intensity. However, water ponding effects were not taken into account during and after the rainfall event in the MCN method. Water that collects in depressions may develop seals that are more impermeable than the exposed soils. While a layer of water protects the soil surface from surface sealing, sediments entering the pool may settle to form highly impermeable depositional seals, thus reducing infiltration during the rainfall event. However, after the rainfall event, water that remains in pools can either infiltrate slowly into the soil or be more rapidly evaporated than water that had infiltrated into soil that is not submergered. The evaporation of water from pools is not identified by measurements of the runoff, but is included in the infiltration. This may have some influence on the steady infiltration determined by the MCN method.

In a region where there is a scarcity of observed data, it would be necessary to use $I_{a-0.25}$ for the MCN method. This value could be obtained by first deriving CN values from the tables given in the National Engineering Handbook (Section 4) (NEH-4) (SCS 1972) and calculating the $S$ value using Eq. (3). Then $I_{a-0.25}$ could be calculated using the statistical relationship of $I_a = 0.2S$, although a loss of accuracy might occur as indicated by the results in this study. In addition, $f_c$ could be derived from the minimum infiltration rate, which could be estimated according to the Hydrologic Soil Group (HSG) used in the CN method based on the different soil textures in the region under study. Moreover, it should be noted that the lack of good quality data for hydrologic modeling in arid regions could have a greater effect on the results than the model performance (Pilgrim et al. 1988).

**Conclusions**

A modified CN (MCN) method was developed by introducing a steady infiltration term into the CN method in order to simulate infiltration after runoff initiation during rainfall events in the Qiaozhi-West watershed on the Loess Plateau, China. The observed rainfall–runoff data of 14 rainfall events in the study watershed were used to determine the watershed final infiltration rate ($f_c$), required by the MCN method, by back-calculating $f_c$ for each event and using the median value. Simulated watershed infiltration after runoff initiation by the MCN and CN methods, using either observed ($I_{a-obs}$) or calculated ($I_{a-0.25}$) initial abstraction, indicated that the MCN method greatly improved the agreement between the observed and simulated infiltration values. For the MCN method, the predicted infiltration determined by using the calculated initial abstraction ($I_{a-0.25}$) was not as accurate as that when using the observed initial abstraction ($I_{a-obs}$), which might be the first choice when the MCN model is applied to other watersheds. In contrast, the CN method using either of the two initial abstractions consistently predicted lower infiltration values than those predicted by the MCN method, and underestimated all the infiltration values for the studied events, especially for the higher infiltration values. Moreover, in a region with observed data scarcity, the use of $I_{a-0.25}$ would be necessary for the MCN method, which still yields more accurate results than the CN method. The watershed runoff was also derived by the MCN and CN methods using only the observed initial abstraction ($I_{a-obs}$). The model simulation performances for both methods were poorer than those when using them to calculate watershed infiltration after runoff initiation. A comparison of the results derived by the two methods concluded that the MCN method had a superior model simulation performance than the CN method. In addition, the MCN method needed only one additional parameter, i.e., the watershed final infiltration rate ($f_c$), than the CN method. Considering both the theoretical analyses and the practical application results, it was concluded that the MCN method should be more appropriate for simulating watershed infiltration after runoff initiation and runoff in the small watersheds on the Loess Plateau. This would be especially useful in arid and semiarid regions, where data availability is often the main obstacle to the application of existing infiltration models.

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**References**


